Autonomous Vehicle Simulation (AVS) Laboratory, University of Colorado

Basilisk Technical Memorandum

Document ID: Basilisk-MRP_Steering

MRP STEERING ADCS CONTROL MODULE

Prepared by	H. Schaub

Status: Initial Documentation
Scope/Contents
This module uses the MRP Steering control logic to determine the ADCS control torque vector $\boldsymbol{L}_r$.

Rev:	**Change Description**	**By**
Draft	Initial Documentation Draft	H. Schaub
1.0	Updated the sign definition of $\boldsymbol{L}_r$	H. Schaub
1.1	Small updates after code review	H. Schaub

Contents

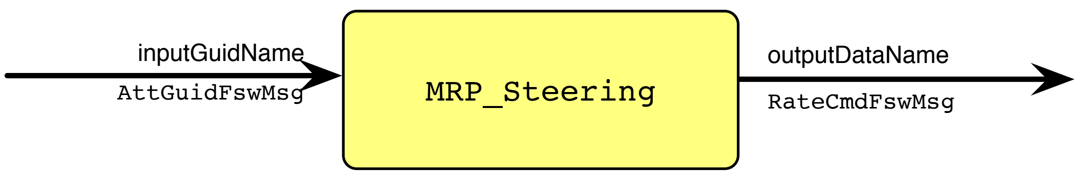


Fig. 1: `MRP_Steering()` Module I/O Illustration

1 Module Overview

The module input and output messages are illustrated in Figure 1. The intend of this module is to implement an MRP steering law where the control output is a vector of commanded body rates. To use this module it is required to use a separate rate tracking servo control module as well.

2 Steering Law Goals

This technical note develops a new MRP based steering law that drives a body frame $\mathcal{B} : \{\hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$ towards a time varying reference frame $\mathcal{R} : \{\hat{\boldsymbol{r}}_1, \hat{\boldsymbol{r}}_2, \hat{\boldsymbol{r}}_3\}$. The inertial frame is given by $\mathcal{N} : \{\hat{\boldsymbol{n}}_1, \hat{\boldsymbol{n}}_2, \hat{\boldsymbol{n}}_3\}$. The RW coordinate frame is given by $\mathcal{W}_{\rangle} : \{\hat{\boldsymbol{g}}_{s_i}, \hat{\boldsymbol{g}}_{t_i}, \hat{\boldsymbol{g}}_{g_i}\}$. Using MRPs, the overall control goal is

$$\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} \rightarrow 0 \tag{1}$$

The reference frame orientation $\boldsymbol{\sigma}_{\mathcal{R}/\mathcal{N}}$, angular velocity $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ and inertial angular acceleration $\dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}}$ are assumed to be known.

The rotational equations of motion of a rigid spacecraft with N Reaction Wheels (RWs) attached are given by[1]

$$[I_{RW}]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}]\left([I_{RW}]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} \tag{2}$$

where the inertia tensor $[I_{RW}]$ is defined as

$$[I_{RW}] = [I_s] + \sum_{i=1}^{N}\left(J_{t_i}\hat{\boldsymbol{g}}_{t_i}\hat{\boldsymbol{g}}_{t_i}^T + J_{g_i}\hat{\boldsymbol{g}}_{g_i}\hat{\boldsymbol{g}}_{g_i}^T\right) \tag{3}$$

The spacecraft inertial without the N RWs is $[I_s]$, while J_{s_i}, J_{t_i} and J_{g_i} are the RW inertias about the body fixed RW axis $\hat{\boldsymbol{g}}_{s_i}$ (RW spin axis), $\hat{\boldsymbol{g}}_{t_i}$ and $\hat{\boldsymbol{g}}_{g_i}$. The $3 \times N$ projection matrix $[G_s]$ is then defined as

$$[G_s] = \left[\cdots {}^{\mathcal{B}}\hat{\boldsymbol{g}}_{s_i} \cdots \right] \tag{4}$$

The RW inertial angular momentum vector $\boldsymbol{h}_s$ is defined as

$$h_{s_i} = J_{s_i}(\omega_{s_i} + \Omega_i) \tag{5}$$

Here Ω_i is the i^{th} RW spin relative to the spacecraft, and the body angular velocity is written in terms of body and RW frame components as

$$\boldsymbol{\omega} = \omega_1 \hat{\boldsymbol{b}}_1 + \omega_2 \hat{\boldsymbol{b}}_2 + \omega_3 \hat{\boldsymbol{b}}_3 = \omega_{s_i} \hat{\boldsymbol{g}}_{s_i} + \omega_{t_i} \hat{\boldsymbol{g}}_{t_i} + \omega_{g_i} \hat{\boldsymbol{g}}_{g_i} \tag{6}$$

3 MRP Steering Law

3.1 Steering Law Stability Requirement

As is commonly done in robotic applications where the steering laws are of the form $\dot{\boldsymbol{x}} = \boldsymbol{u}$, this section derives a kinematic based attitude steering law. Let us consider the simple Lyapunov candidate function[1,2]

$$V(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = 2 \ln \left(1 + \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} \right) \tag{7}$$

in terms of the MRP attitude tracking error $\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}$. Using the MRP differential kinematic equations

$$\dot{\boldsymbol{\sigma}}_{\mathcal{B}/\mathcal{R}} = \frac{1}{4}[B(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}})]^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}} = \frac{1}{4}\left[(1 - \sigma_{\mathcal{B}/\mathcal{R}}^2)[I_{3\times3} + 2[\tilde{\boldsymbol{\sigma}}_{\mathcal{B}/\mathcal{R}}] + 2\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \right] {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}} \tag{8}$$

where $\sigma_{\mathcal{B}/\mathcal{R}}^2 = \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}$, the time derivative of V is

$$\dot{V} = \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \left({}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{R}} \right) \tag{9}$$

To create a kinematic steering law, let $\mathcal{B}^*$ be the desired body orientation, and $\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}}$ be the desired angular velocity vector of this body orientation relative to the reference frame $\mathcal{R}$. The steering law requires an algorithm for the desired body rates $\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}}$ relative to the reference frame make $\dot{V}$ in Eq. (9) negative definite. For this purpose, let us select

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}^*/\mathcal{R}} = -\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) \tag{10}$$

where $\boldsymbol{f}(\boldsymbol{\sigma})$ is an even function such that

$$\boldsymbol{\sigma}^T \boldsymbol{f}(\boldsymbol{\sigma}) > 0 \tag{11}$$

The Lyapunov rate simplifies to the negative definite expression:

$$\dot{V} = -\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}^T \boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) < 0 \tag{12}$$

3.2 Saturated MRP Steering Law

A very simple example would be to set

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = K_1 \boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} \tag{13}$$

where $K_1 > 0$. This yields a kinematic control where the desired body rates are proportional to the MRP attitude error measure. If the rate should saturate, then $\boldsymbol{f}()$ could be defined as

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = \begin{cases} K_1\sigma_i & \text{if } |K_1\sigma_i| \leqslant \omega_{\text{max}} \\ \omega_{\text{max}}\text{sgn}(\sigma_i) & \text{if } |K_1\sigma_i| > \omega_{\text{max}} \end{cases} \tag{14}$$

where

$$\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} = (\sigma_1, \sigma_2, \sigma_3)^T$$

A smoothly saturating function is given by

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = \arctan\left(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}\frac{K_1\pi}{2\omega_{\text{max}}}\right)\frac{2\omega_{\text{max}}}{\pi} \tag{15}$$

where

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) = \begin{pmatrix} f(\sigma_1) \\ f(\sigma_2) \\ f(\sigma_3) \end{pmatrix} \tag{16}$$

Here as $\sigma_i \to \infty$ then the function f smoothly converges to the maximum speed rate $\pm\omega_{\text{max}}$. For small $|\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}|$, this function linearizes to

$$\boldsymbol{f}(\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}) \approx K_1\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}} + \text{ H.O.T} \tag{17}$$

If the MRP shadow set parameters are used to avoid the MRP singularity at 360°, then $|\boldsymbol{\sigma}_{\mathcal{B}/\mathcal{R}}|$ is upper limited by 1. To control how rapidly the rate commands approach the ω_{max} limit, Eq. (15) is modified to include a cubic term:

$$f(\sigma_i) = \arctan\left((K_1\sigma_i + K_3\sigma_i^3)\frac{\pi}{2\omega_{\text{max}}}\right)\frac{2\omega_{\text{max}}}{\pi} \tag{18}$$

The order of the polynomial must be odd to keep $\boldsymbol{f}()$ an even function. A nice feature of Eq. (18) is that the control rate is saturated individually about each axis. If the smoothing component is removed to reduce this to a bang-band rate control, then this would yield a Lyapunov optimal control which minimizes $\dot{V}$ subject to the allowable rate constraint ω_{max}.

Figure 2 illustrates how the parameters ω_{max}, K_1 and K_3 impact the steering law behavior. The maximum steering law rate commands are easily set through the ω_{max} parameters. The gain K_1 controls the linear stiffness when the attitude errors have become small, while K_3 controls how rapidly the steering law approaches the speed command limit.

The required velocity servo loop design is aided by knowing the body-frame derivative of ${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ to implement a feed-forward components. Using the $\boldsymbol{f}()$ function definition in Eq. (16), this requires the time derivatives of $f(\sigma_i)$.

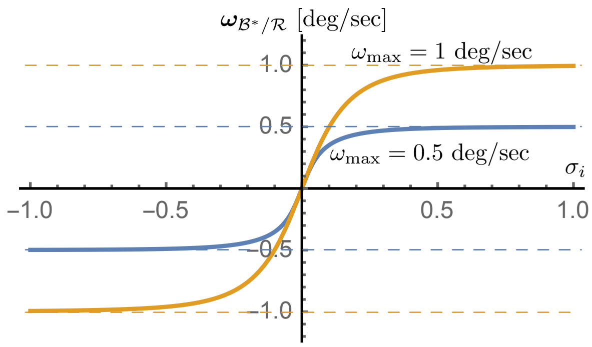
$$\frac{{}^{\mathcal{B}}\text{d}({}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}})}{\text{d}t} = \boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} = -\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{\sigma}_{\mathcal{B}*/\mathcal{R}}}\dot{\boldsymbol{\sigma}}_{\mathcal{B}*/\mathcal{R}} = -\begin{pmatrix} \frac{\partial f}{\partial\sigma_1}\dot{\sigma}_1 \\ \frac{\partial f}{\partial\sigma_2}\dot{\sigma}_2 \\ \frac{\partial f}{\partial\sigma_3}\dot{\sigma}_3 \end{pmatrix} \tag{19}$$

where

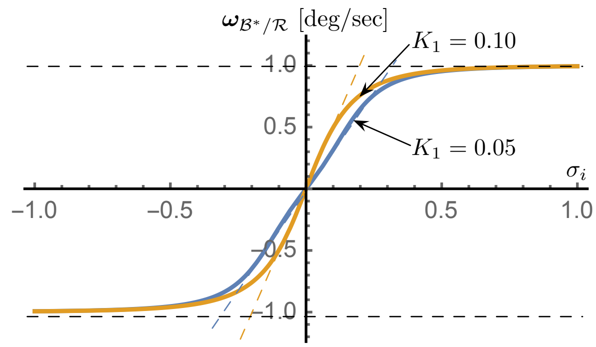
$$\dot{\boldsymbol{\sigma}}_{\mathcal{B}*/\mathcal{R}} = \begin{pmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \\ \dot{\sigma}_3 \end{pmatrix} = \frac{1}{4}[B(\boldsymbol{\sigma}_{\mathcal{B}*/\mathcal{R}})]^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} \tag{20}$$

Using the general $f()$ definition in Eq. (18), its sensitivity with respect to σ_i is

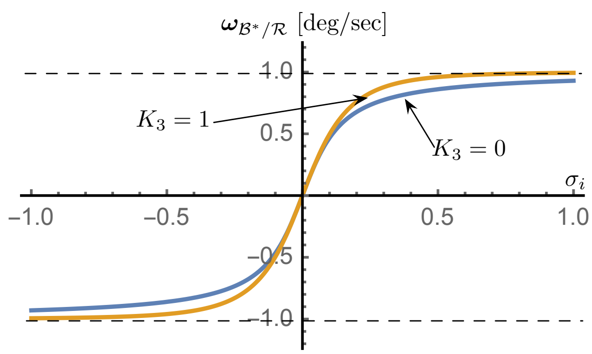
$$\frac{\partial f}{\partial\sigma_i} = \frac{(K_1 + 3K_3\sigma_i^2)}{1 + (K_1\sigma_i + K_3\sigma_i^3)^2\left(\frac{\pi}{2\omega_{\text{max}}}\right)^2} \tag{21}$$



(a) ω_{max} dependency with $K_1 = 0.1$, $K_3 = 1$



(b) K_1 dependency with $\omega_{\text{max}} = 1^\circ/s$, $K_3 = 1$



(c) K_3 dependency with $\omega_{\text{max}} = 1^\circ/s$, $K_1 = 0.1$

Fig. 2: Illustrations of MRP Steering Parameters Influence.

4 Angular Velocity Servo Sub-System

To implement the kinematic steering control, a servo sub-system must be included which will produce the required torques to make the actual body rates track the desired body rates. The following development is an illustration of such a servo control module, such as the `rateServoFullNonlinear` module. However, other body rate tracking control modules could be used as well.

The angular velocity tracking error vector is defined as

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{B}*} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} \tag{22}$$

where the $\mathcal{B}^*$ frame is the desired body frame from the kinematic steering law. Note that

$$\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{N}} = \boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}} + \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{23}$$

where $\boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$ is obtained from the attitude navigation solution, and $\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ is the kinematic steering rate command. To create a rate-servo system that is robust to unmodeld torque biases, the state $\boldsymbol{z}$ is defined as:

$$\boldsymbol{z} = \int_{t_0}^{t_f} {}^{\mathcal{B}}\delta\boldsymbol{\omega} \ \mathrm{d}t \tag{24}$$

The rate servo Lyapunov function is defined as

$$V_{\boldsymbol{\omega}}(\delta\boldsymbol{\omega}, \boldsymbol{z}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I_{\text{RW}}]\delta\boldsymbol{\omega} + \frac{1}{2}\boldsymbol{z}^T[K_I]\boldsymbol{z} \tag{25}$$

where the vector $\delta\boldsymbol{\omega}$ and tensor $[I_{\text{RW}}]$ are assumed to be given in body frame components, $[K_i]$ is a symmetric positive definite matrix. The time derivative of this Lyapunov function is

$$\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T \left([I_{\text{RW}}]\delta\boldsymbol{\omega}' + [K_I]\boldsymbol{z}\right) \tag{26}$$

Using the identities $\boldsymbol{\omega}'_{\mathcal{B}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ and $\boldsymbol{\omega}'_{\mathcal{R}/\mathcal{N}} = \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}$,[1] the body frame derivative of $\delta\boldsymbol{\omega}$ is

$$\delta\boldsymbol{\omega}' = \dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} - \boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} - \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}} \tag{27}$$

Substituting Eqs. (2) and (27) into the $\dot{V}_{\boldsymbol{\omega}}$ expression in Eq. (26) yields

$$\begin{aligned}\dot{V}_{\boldsymbol{\omega}} = \delta\boldsymbol{\omega}^T \Big(&- [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) - [G_s]\boldsymbol{u}_s + \boldsymbol{L} + [K_I]\boldsymbol{z} \\ &- [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}})\Big)\end{aligned} \tag{28}$$

Let $[P]^T = [P] >$ be a symmetric positive definite rate feedback gain matrix. The servo rate feedback control is defined as

$$\begin{aligned}[G_s]\boldsymbol{u}_s = [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}] \left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \\ - [I_{\text{RW}}](\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}} + \dot{\boldsymbol{\omega}}_{\mathcal{R}/\mathcal{N}} - \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\omega}_{\mathcal{R}/\mathcal{N}}) + \boldsymbol{L}\end{aligned} \tag{29}$$

Defining the right-hand-side as $\boldsymbol{L}_r$, this is rewritten in compact form as

$$[G_s]\boldsymbol{u}_s = -\boldsymbol{L}_r \tag{30}$$

The array of RW motor torques can be solved with the typical minimum norm inverse

$$\boldsymbol{u}_s = [G_s]^T \left([G_s][G_s]^T\right)^{-1} (-\boldsymbol{L}_r) \tag{31}$$

To analyze the stability of this rate servo control, the $[G_s]\boldsymbol{u}_s$ expression in Eq. (29) is substituted into the Lyapunov rate expression in Eq. (28).

$$\begin{aligned}\dot{V}_\omega &= \delta\boldsymbol{\omega}^T \Big(- [P]\delta\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) + [\tilde{\boldsymbol{\omega}}_{\mathcal{B}*/\mathcal{N}}] \left([I_{\text{RW}}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \Big) \\ &= \delta\boldsymbol{\omega}^T \Big(- [P]\delta\boldsymbol{\omega} - [\widetilde{\delta\boldsymbol{\omega}}] \left([I_{RW}]\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} + [G_s]\boldsymbol{h}_s\right) \Big) \\ &= -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} < 0\end{aligned} \tag{32}$$

Thus, in the absence of unmodeled torques, the servo control in Eq. (29) is asymptotically stabilizing in rate tracking error $\delta\boldsymbol{\omega}$.

Next, the servo robustness to unmodeled external torques is investigated. Let us assume that the external torque vector $\boldsymbol{L}$ in Eq. (2) only approximates the true external torque, and the unmodeled component is given by $\Delta\boldsymbol{L}$. Substituting the true equations of motion and the same servo control in Eq. (29) into the Lyapunov rate expression in Eq. (26) leads to

$$\dot{V}_\omega = -\delta\boldsymbol{\omega}^T[P]\delta\boldsymbol{\omega} - \delta\boldsymbol{\omega}^T\Delta\boldsymbol{L} \tag{33}$$

This $\dot{V}_\omega$ is no longer negative definite due to the underdetermined sign of the $\delta\boldsymbol{\omega}^T\Delta\boldsymbol{L}$ components. Equating the Lyapunov rates in Eqs. (26) and (33) yields the following servo closed loop dynamics:

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}' + [P]\delta\boldsymbol{\omega} + [K_I]\boldsymbol{z} = \Delta\boldsymbol{L} \tag{34}$$

Assuming that $\Delta\boldsymbol{L}$ is either constant as seen by the body frame, or at least varies slowly, then taking a body-frame time derivative of Eq. (34) is

$$[I_{\text{RW}}]\delta\boldsymbol{\omega}'' + [P]\delta\boldsymbol{\omega}' + [K_I]\delta\boldsymbol{\omega} = \Delta\boldsymbol{L}' \approx 0 \tag{35}$$

As $[I_{\text{RW}}]$, $[P]$ and $[K_I]$ are all symmetric positive definite matrices, these linear differential equations are stable, and $\delta\boldsymbol{\omega} \to 0$ given that assumption that $\Delta\boldsymbol{L}' \approx 0$.

5 Testing

Two tests are provided with this module. The first is a unit test that compares the computed $\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ and $\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}}$ to truth values computed in the python unit test. The second is an integrated test of this module with `rateServoFullNonlin` as well, comparing the desired torques computed $\boldsymbol{L}_r$ with truth values computed in the test. Both tests check a set of gains $K1, K3$ and ω_{max} on a rigid body with no external torques, and with a fixed input reference attitude message. The torque requested by the controller is evaluated against python computed torques at 0s, 0.5s, 1s, 1.5s and 2s to within a tolerance of 10^{-12} for the integrated test.

- The test is run for a case with $K1 =$ 0 or 0.15
- The gain $K3$ is set to 0 or 1
- The saturation rate ω_{max} is set to 1.5 degrees/second or 0.001 degree/second

All permutations of these test cases are expected to pass. The rate servo module `rateServoFullNonlin` has dedicated unit tests to check various parameters required there, including integral gain on/off, presence of external torques and other variables.

6 User's guide

The following variables are required for this module:

- The gains $K1, K3$
- The value of ω_{max}

This module returns the values of $\boldsymbol{\omega}_{\mathcal{B}*/\mathcal{R}}$ and $\boldsymbol{\omega}'_{\mathcal{B}*/\mathcal{R}}$, which are used in the rate servo-level controller to compute required torques.

The control update period Δt is evaluated automatically.

REFERENCES

[1] Hanspeter Schaub and John L. Junkins. *Analytical Mechanics of Space Systems*. AIAA Education Series, Reston, VA, 3rd edition, 2014.

[2] Panagiotis Tsiotras. New control laws for the attitude stabilization of rigid bodies. In *13th IFAC Symposium on Automatic Control in Aerospace*, pages 316–321, Palo Alto, CA, Sept. 12–16 1994.

[3] Peter Carlisle Hughes. *Spacecraft Attitude Dynamics*. John Wiley & Sons, Inc., New York, 1986.