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Abstract

The relative motion dynamics of a Coulomb satellite formation is considered. Electrostatic forces are exploited to control the relative motion. The formation angular momentum about an inertial point of reference and the formation center of mass are studied. The insights provide constraints on the relative motion description for both inertial and relative coordinates. The inertial formation center of mass motion is studied in more detail and compared to Keplerian motion. Finally, a coordinate frame is introduced which tracks the principal axes of the formation. This allows a convenient method to describe the formation shape at large.

Introduction

An important concern with spacecraft formation flying is how to control the relative motion. Since the formation is expected to have a life time of several years, a fuel- and power-efficient solution is crucial. For the sparse radar aperture missions, the spacecraft are to fly on relative orbits ranging from 0.5 to 10 kilometers. For these missions fuel efficient ion engines are being considered to stabilize the relative orbits and avoid secular drift among the satellites. Recently a novel formation flying concept using electrostatic propulsion has been proposed in References 5 and 14. The charge of the spacecraft is controlled to generate inter-spacecraft Coulomb forces. Such forces can be used to attract or repel the craft of each other, and thus control their relative motion. Studying the electrostatic charging data of the geostationary SCATHA spacecraft,[7] it became evident that it is possible to generate forces of the order of 10–1000 μN. The values are comparable to those of ion engines being considered to control current formation flying concepts. What was inspiring was that virtually zero fuel mass would be consumed to generate these forces, and the electrical power requirements were typically around 1 Watt or less. However, since the electric field strength will drop off with the inverse square of the separation distance, this relative motion control concept is only viable for relatively tight formations with 10-100 meters separation distances. Beyond such distances, electrostatic discharge among spacecraft components becomes an increasing concern. Further, in a space plasma environment, the electric field strength drops off in an exponential manner due to the presence of charged plasma particles. The severity of the exponential drop off is expressed through the Debye length.[1,9] For low Earth orbits (LEO), the Debye length is of the order of centimeters, making the Coulomb formation flying concept impractical. However, extensive data is available for the plasma environment at geostationary orbits (GEO), where the Debye length can vary between 100–1400 m. Thus, at GEO altitudes or higher, the Coulomb formation flying concept appears feasible.

This essentially propellantless mode of propulsion comes at the price of a greatly increased level of complexity of the relative motion dynamics. The electrostatic fields directly tie together and couple the motion of *all* charged satellites. If one charged satellite were to change its position, then the motion of all other charged satellites in the formation will be affected. This paper attempts to shed some light on how the Coulomb formation will evolve as an entity, as opposed to individual satellite motion. To do so, the angular momentum of the formation about the Earth center (inertial point) and the formation center of mass is examined. Each craft is assumed to be subjected to the standard inverse square gravitational attraction, as well as the Coulomb forces from all other craft in the formation. Studying the inertial angular momentum vector, conclusions can be drawn regarding the controllability of the entire formation. Of particular interest is that given a desired Coulomb formation, can a feedback law be developed that will achieve this formation given any arbitrary initial formation errors?

To be able to describe the overall formation motion, a formation coordinate frame is considered. The formation is no longer treated as a series of individual, un-coupled satellites, but rather as a fluid-like entity subjected to orbital dynamics. The inter-spacecraft Coulomb forces are similar to fluid internal forces that can be used to control the shape and size of the formation. As the formation rotates and moves, the formation body frame will move in an analogous manner. Studying the formation frame orientation, it is possible to study the overall formation attitude change, as well as possibly control it. Cochran et al. discuss in Reference 2 how to model a spacecraft formation as a rigid body. Here the spacecraft are assumed to be in fixed positions relative to each other, and thus form a discretely distributed *rigid* spacecraft structure. The formulation adopted in this paper allows for all spacecraft to move relative to each other, and thus act more as a fluid than a rigid body. However, given an appropriate control, it might be possible to "freeze" this fluid and have all craft form a rigid formation as shown in Reference.[6] Sengupta and Vadali also employ a rigid body analogy to describe relative motion in Reference 16, but use it to describe the motion of the individual satellites, and not to describe the formation shape and size as a whole.

Lastly, the formation center of mass motion is studied in detail, since this location forms the origin of the proposed formation body frame. In previous formation flying control work, the satellite motion is defined to be relative to some chief point.[4,8,12,15] For example, this point could be the geometric center of the formation, or the center of mass, or simply another satellite of the formation. In the Coulomb feedback control laws presented in References 14 and 11, all relative motion is written explicitly with respect to the formation center of mass motion, which is assumed to perform a Keplerian motion. Here the validity of Keplerian formation center of mass motion is studied in greater detail. Both an analytical and numerical analysis are presented.

Coulomb Formation Equations of Motion

Let the Coulomb formation consist of N satellites of different mass m_i. Let the formation chief position $\boldsymbol{r}_c$ be defined as the inertial formation center of mass A vector $\boldsymbol{r}_i$ will denote a satellite position vector relative to the inertial center of Earth point, while a $\boldsymbol{\rho}_i$ vector will denote a relative position vector with respect to the chief position. Typically the motion of the Coulomb satellites is expressed relative to the center of mass motion of the formation. These differences could be Cartesian coordinate where the relative position vector $\boldsymbol{\rho}_i$ components are expressed with respect to the rotating chief Hill frame $\mathcal{H}$ as ${}^{\mathcal{H}}\boldsymbol{\rho}_i = \begin{pmatrix} x & y & z \end{pmatrix}^T$. An alternate coordinate choice would be to express the relative motion in terms of orbit element differences.[11,14] Given the various satellite masses m_i, the center of mass $\boldsymbol{r}_c$ is computed using[13]

$$\boldsymbol{r}_c = \frac{1}{M}\left(\sum_{k=1}^{N} m_i \boldsymbol{r}_i\right) \tag{1}$$

where $M = \sum_{k=1}^{N} m_i$ is the total formation mass. The Coulomb force that craft j exerts onto craft i is given by

$$\boldsymbol{f}_{ij} = k_c \frac{\boldsymbol{r}_{ji}}{r_{ji}^3} q_i q_j e^{-\frac{r_{ji}}{\lambda_d}} \tag{2}$$

where $\boldsymbol{r}_{ji} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $|\boldsymbol{r}_{ji}| = r_{ji} = r_{ij}$, $k_c = 8.99 \cdot 10^9$ Nm^2/C^2 is Coulomb's constant, and λ_d is the plasma Debye length. Assuming a standard inverse square gravitational attraction and infinite λ_d, the inertial equations of motion of the i-th craft are

$$\ddot{\boldsymbol{r}}_i + \frac{\mu}{r_i^3}\boldsymbol{r}_i = \frac{1}{m_i}\sum_{j=1}^{N} \boldsymbol{f}_{ij} \qquad \text{for } i \neq j \tag{3}$$

Note that these equations of motion are valid for any conic-section orbit type. Previous research into the Coulomb formation dynamics has focused on using the CW, which linearize the relative motion dynamics assuming the chief motion is circular:[5]

$$\begin{pmatrix} \ddot{x}_i - 2n\dot{y}_i - 3n^2 x_i \\ \ddot{y}_i + 2n\dot{x}_i \\ \ddot{z}_i + nz_i \end{pmatrix} = \frac{k_c}{m_i}\sum_{j=1}^{N} \frac{\boldsymbol{r}_{ji}}{r_{ji}^3} q_i q_j \tag{4}$$

These equations are very convenient for typical formation flying studies since they express the relative motion dynamics directly in terms of the relative position vector components. However, to study angular momentum conservation laws, they are not very convenient since the vector components are taken with respect to the rotating chief Hill frame. Instead, the full nonlinear equations of Eq. (3) will be used instead.

Coulomb Formation Conservation Laws

Formation Angular Momentum about Inertial Point

To study the overall motion of the Coulomb formation as a whole, let us examine the angular momentum vector of the entire formation. The moments are first taken relative to the Earth's center, which is assumed to be an inertial point for the purpose of this study. The inertial angular momentum vector $\boldsymbol{H}_e$ is then written as

$$\boldsymbol{H}_e = \sum_{i=1}^{N} \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i \tag{5}$$

The gravitational acceleration vector is known to not cause any change in the inertial momentum vector, because its direction is collinear with the satellite position vector. Since the Coulomb forces are all formation internal forces, they too cannot cause the inertial momentum vector to change. Thus we find $\dot{\boldsymbol{H}}_e = 0$. This angular momentum vector of the entire formation thus provides three constraint equations that the Coulomb satellite motion must abide by. If a system of 3 Coulomb satellites is studied through 3 inertial position vectors, then there would be 9 degrees of freedom for the entire formation. However, due to the angular momentum constraint, three degrees of freedom would be lost, leaving only 6 unconstrained degrees of freedom.

Any Coulomb control strategy must take these formation constraints into account. It may be possible that not all craft in the Coulomb formation need to achieve a specific trajectory. Additional craft may be introduced specifically to facilitate the controllability of the true sensor craft. The other craft would absorb the inertial angular momentum difference between current and desired formation and make sure that the inertial angular momentum constraints are satisfied. Also, missions are envisioned where the precise relative motion may not need to be controlled. For example, it might be sufficient to define the desired relative motion to be a formation with equal spacecraft spacing (equilateral triangle for the planar three craft case).

Formation Angular Momentum about Center of Mass

The inertial angular momentum expression in Eq. (5) is written in terms of the inertial position vectors $\boldsymbol{r}_i$. Because it is preferred to write the Coulomb satellite motion as coordinate differences relative to the center of mass motion (chief), we write $\boldsymbol{r}_i = \boldsymbol{r}_c + \boldsymbol{\rho}_i$. The inertial angular momentum vector is written as[13]

$$\boldsymbol{H}_e = \boldsymbol{r}_c \times M\dot{\boldsymbol{r}}_c + \sum_{i=1}^{N} \boldsymbol{\rho}_i \times m_i \dot{\boldsymbol{\rho}}_i = \boldsymbol{r}_c \times M\dot{\boldsymbol{r}}_c + \boldsymbol{H}_c \tag{6}$$

where $\boldsymbol{H}_c$ is the formation angular momentum about the center of mass. Note that this expression is analogous to that used to express the angular momentum of a rigid body in space. Because $\dot{\boldsymbol{H}}_e = 0$ for a Coulomb formation, by differentiating Eq. (6) we find that

$$\dot{\boldsymbol{H}}_c = -\boldsymbol{r}_c \times M\ddot{\boldsymbol{r}}_c = -\boldsymbol{r}_c \times \boldsymbol{F}_c \tag{7}$$

where $\boldsymbol{F}_c = M\ddot{\boldsymbol{r}}_c$ is the effective force acting on the formation center of mass. If the Coulomb formation is floating in space without any external forces acting on it (i.e. far from Earth), then $\ddot{\boldsymbol{r}}_c = 0$ and $\boldsymbol{H}_c$ would be conserved. However, if the formation is orbiting about a planet and subject to an inverse square gravity field, then an external torque similar to the rigid body gravity gradient torque will act on $\boldsymbol{H}_c$.

Using Eqs. (1), (3) and $\boldsymbol{f}_{ij} = -\boldsymbol{f}_{ji}$, the effective force vector $\boldsymbol{F}_c$ can be written as

$$\boldsymbol{F}_c = M\ddot{\boldsymbol{r}}_c = \sum_{i=1}^{N} m_i \ddot{\boldsymbol{r}}_i = \sum_{i=1}^{N} \left(-m_i \frac{\mu}{r_i^3}\boldsymbol{r}_i + \sum_{j=1}^{N} \boldsymbol{f}_{ij} \right) = -\sum_{i=1}^{N} m_i \frac{\mu}{r_i^3}\boldsymbol{r}_i \tag{8}$$

Note that this expression is not very convenient yet, since it requires us to know the inertial position vectors $\boldsymbol{r}_i$ of each satellite. It is preferred to be able to express the Coulomb satellite motion using both the center of mass motion $\boldsymbol{r}_c$ and the relative position vectors $\boldsymbol{\rho}_i$. Further, the ρ values for Coulomb formations are typically 10-100 meters, which is very small compared to the inertial center of mass orbit radius r_c. Using $\boldsymbol{r}_i = \boldsymbol{r}_c + \boldsymbol{\rho}_i$, the $1/r_i^3$ term is approximated using a standard truncated binomial expansion. Only retaining the first order terms and making use of the center of mass definition, this expansion allows us to approximate $\boldsymbol{F}_c$ as

$$\boldsymbol{F}_c = -M\frac{\mu}{r_c^3}\boldsymbol{r}_c + 3\frac{\mu}{r_c^5}\left[\sum_{i=1}^{N}(m_i\boldsymbol{\rho}_i \times (\boldsymbol{\rho}_i \times \boldsymbol{r}_c)) + \left(\sum_{i=1}^{N} m_i \rho_i^2 \right) \boldsymbol{r}_c \right] \tag{9}$$

If the systems of particles were a rigid body, then the inertia matrix would be defined as

$$[I] = -\sum_{i=1}^{N} m_i [\tilde{\boldsymbol{\rho}}_i][\tilde{\boldsymbol{\rho}}_i] \tag{10}$$

where the matrix tilde operator is is equivalent to $\boldsymbol{x} \times \boldsymbol{y}$. The identical inertia matrix definition is used here to define the Coulomb formation inertia $[I]$ about the formation center of mass. Whereas the $\boldsymbol{\rho}_i$ position vectors would be fixed relative to a body fixed frame, these relative position vectors will be time varying for Coulomb satellites. Thus, instead of describing the formation as a rigid body, it is described as a fluid-like entity. This formation inertia matrix definition $[I]$ allows us to write the center of mass force vector $\boldsymbol{F}_c$ as the matrix equation

$$\boldsymbol{F}_c = -M\frac{\mu}{r_c^3}\boldsymbol{r}_c - 3\frac{\mu}{r_c^5}\left([I]\boldsymbol{r}_c - \left(\sum_{i=1}^{N} m_i \rho_i^2 \right) \boldsymbol{r}_c \right) \tag{11}$$

Substituting Eq. (11) into Eq. (7) and carrying out the vector cross products, we find the formation angular momentum rate expression approximated as

$$\dot{\boldsymbol{H}}_c = -\boldsymbol{r}_c \times \boldsymbol{F}_c = 3\frac{\mu}{r_c^5}[\tilde{\boldsymbol{r}}_c][I]\boldsymbol{r}_c = \boldsymbol{\tau}_c \tag{12}$$

If the masses m_i were part of a rigid body, then $\boldsymbol{\tau}_c$ would be referred to as the gravity gradient torque. It arises from the fact that if two equal masses are at different orbit radii, then the inverse square gravity field will exert a larger force onto the mass with the smaller orbit radius.

Thus, while the formation angular momentum about an inertial point (Earth's center) was found to be conserved for Coulomb satellites, the formation angular momentum about the formation center of mass is generally not conserved in the formation. An upper bound on the gravity gradient torque is found by assuming that all of the formation masses m_i are distributed at the outer formation radius ρ.

$$\tau \leq \frac{\mu M}{r_c}\left(\frac{\rho}{r_c}\right)^2 \tag{13}$$

As r_c grows sufficiently large, the gravity gradient torque will become very small and could be neglected from a formation analysis. Note that the three constraints of the $\dot{\boldsymbol{H}}_e$ or $\dot{\boldsymbol{H}}_c$ formulation are essentially equivalent. There are still only three momentum constraints that a Coulomb formation must satisfy. The only difference is that the gravity gradient torque $\boldsymbol{\tau}_c$ has been approximated through a truncated series.

Formation Center of Mass Motion

If the Coulomb satellite motion is expressed relative to the center of mass through $\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{r}_c$, the center of mass motion definition in Eq. (1) can be written in terms of $\boldsymbol{\rho}_i$ as

$$\sum_{k=1}^{N} m_i \boldsymbol{\rho}_i = 0 \tag{14}$$

This equation provides three additional constraints on the Coulomb satellite motion $\boldsymbol{\rho}_i$ that must be satisfied at all time *if* the satellite motion is expressed relative to the formation center of mass. Combined with the three inertial angular momentum constraints, there are a total of six constraints on the Coulomb satellite motion. If inertial position vectors are used, then the center of mass constraint does not appear. For example, consider the 2-satellite Coulomb formation discussed in Reference 14. Expressing the satellite motion relative to the formation center of mass, the $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ and $\boldsymbol{r}_c$ vectors have 9 degrees of freedom. With the 6 constraints found for a Coulomb formation, this results in the two Coulomb satellite system being a three degree of freedom system. As the Coulomb formation grows in its number of associated craft, these constraints will become less restrictive. A three craft formation would have 6 degrees of freedom. Note that this degree of freedom discussion does not apply to classical spacecraft formation concepts where each craft is controlled individually. Assuming general three-dimensional thrusting capability, here there are no constraints as to how each satellite can move.

With the formation equations of motion written relative to the formation center of mass, it is important to understand how this point will move in space. Previous work has treated the chief point to be rotating at a constant rate for the near-circular chief orbit case,[6] or that the chief orbit elements are constant.[11,14] Since $M\ddot{\boldsymbol{r}}_c = \boldsymbol{F}_c$, the differential equations of the center of mass are found using Eq. (8):

$$\ddot{\boldsymbol{r}}_c = -\frac{1}{M}\sum_{k=1}^{N} m_i \frac{\mu}{r_i^3}\boldsymbol{r}_i \tag{15}$$

Note that these are the true nonlinear differential equations of $\boldsymbol{r}_c$. The Coulomb forces do not appear explicitly here. Assuming that ρ_i is much smaller than r_c, this differential equation can be approximated using Eq. (11).

$$\ddot{\boldsymbol{r}}_c = -\frac{\mu}{r_c^3}\left[1 + 3\frac{1}{Mr_c^2}\left([I] - [I_{3\times3}]\left(\sum_{i=1}^{N} m_i \rho_i^2\right)\right)\right]\boldsymbol{r}_c \tag{16}$$

Note that the first term in the square brackets above is the standard Keplerian gravitational attraction term. The second term is the first order approximation of the rotation to translation coupling. In rigid body attitude and orbital translational motion, the coupling of the attitude on the translation motion is typically treated as a negligible term. However, typical spacecraft do not have dimensions on the order of 100s of meters. Treating the Coulomb formation as a discretely distributed fluid-like body, we need to investigate if this second term inside the square brackets can be neglected. Assume the formation has an outer radius of ρ, and that all the mass is distributed at this radius. We can provide a conservative upper bound on the term using

$$3\frac{1}{Mr_c^2}\left([I] - [I_{3\times3}]\left(\sum_{i=1}^{N} m_i \rho_i^2\right)\right) \leq 3\frac{\left(\sum_{k=1}^{N} m_i \rho_i^2\right)}{Mr_c^2} = 3\frac{\rho^2}{r_c^2} \tag{17}$$

Assume the Coulomb formation has an outer radius of 100 meters and is at GEO, then the upper bound shown in Eq. (17) is $1.7 \cdot 10^{-11}$. The coupling term of the formation motion on the center of mass motion is about 11 orders of magnitude smaller than the Keplerian gravitational attraction. Thus, even though Coulomb formations would be much larger than typical rigid bodies, it is reasonable to ignore the formation motion influence on the center of mass motion and treat the formation center of mass motion as Keplerian. Note that this conclusion is valid regardless if the chief motion is circular or elliptical in nature.

Formation Body Frame

To describe the overall motion of the formation, the cluster of spacecraft is treated as a distributed system of discrete masses, inter-connected by Coulomb forces. In essence, the formation is treated as a virtual structure with a fluid-like behavior. By designing appropriate spacecraft charging control laws, it can be made to change shape, size and orientation. Instead of describing the motion of each craft within the clusters, statistical measures are used to describe the overall motion. For this purpose, a coordinate frame is associated to the Coulomb formation. Let this formation coordinate frame be called $\mathcal{B} = \{\boldsymbol{r}_c : \hat{\boldsymbol{b}}_1, \hat{\boldsymbol{b}}_2, \hat{\boldsymbol{b}}_3\}$. Its origin is the formation center of mass location $\boldsymbol{r}_c$, while it's orientation is determined by the formation dynamics and overall shape.

Assume that the formation inertia matrix ${}^{\mathcal{B}}[I]$ in Eq. (10) is computed with vector components taken with respect to the rotating chief Hill frame $\mathcal{H}$. The formation body frame $\mathcal{B}$ is defined such that it will diagonalize this formation inertia matrix. Thus, the $\mathcal{B}$ frame orientation axes $\hat{\boldsymbol{b}}_i$ are the principal axes of the $\mathcal{H}$ frame centric inertia matrix. Let $[BH]$ be a rotation matrix that will transform $\mathcal{H}$ frame vectors into $\mathcal{B}$ frame vectors. The formation inertia matrix is written in the $\mathcal{B}$ frame as[13]

$$ {}^{\mathcal{B}}[I] = \text{diag}(I_1, I_2, I_3) = [BH] \cdot {}^{\mathcal{H}}[I] \cdot [BH]^T \tag{18} $$

Since $[I]$ is a symmetric, positive definite matrix, the $[BH]$ matrix is the transpose of the eigenvector matrix of $[I]$, while the principal inertias are the associated eigenvalues of $[I]$.[13] This frame $\mathcal{B}$ and the associated principal formation inertias I_i are very useful to describe the overall shape and orientation of the formation. For example, if two I_i values are nearly equal, then the formation has a near-cylindrical shape. If all three I_i values are identical, then the formation has become spherical. If one I_i is much smaller than the other two, then the formation is performing near-planar relative motion. The attitude matrix $[BH]$ would provide information regarding the orientation of this relative motion plane.

Computing ${}^{\mathcal{H}}[I]$ at every time step, it is possible to compute the corresponding attitude matrix $[BH]$ by solving the numerical eigenvalue/eigenvector problem, or using a singular value decomposition routine. However, these routines have ambiguities regarding the direction of the unit eigenvectors. Care must be taken that the three eigenvectors of ${}^{\mathcal{H}}[I]$ form a proper, right-handed coordinate system.

An alternate approach is that the formation eigenvalue/eigenvector problem could be solved once to setup the initial formation principal inertias and attitude, and then use associated differential equations to propagate these parameters. Let $\boldsymbol{\omega} = \boldsymbol{\omega}_{\mathcal{B}/\mathcal{H}}$ for this discussion. Using $[\dot{BH}] = -[\tilde{\boldsymbol{\omega}}][BH]$, taking the derivative of ${}^{\mathcal{H}}[I]$ we find

$$ \begin{aligned} {}^{\mathcal{H}}[\dot{I}] &= [\dot{BH}]^T \, {}^{\mathcal{B}}[I] \, [BH] + [BH]^T \, {}^{\mathcal{B}}[\dot{I}][BH] + [BH]^T \, {}^{\mathcal{B}}[I][\dot{BH}] \\ &= [BH]^T \left([\tilde{\boldsymbol{\omega}}]^{\mathcal{B}}[I] + {}^{\mathcal{B}}[\dot{I}] - {}^{\mathcal{B}}[I][\tilde{\boldsymbol{\omega}}] \right) [BH] \end{aligned} \tag{19} $$

This leads to the condition

$$ [\tilde{\boldsymbol{\omega}}]^{\mathcal{B}}[I] + {}^{\mathcal{B}}[\dot{I}] - {}^{\mathcal{B}}[I][\tilde{\boldsymbol{\omega}}] = [BH]^{\mathcal{H}}[\dot{I}][BH]^T = [\xi] \tag{20} $$

Since $[\tilde{\boldsymbol{\omega}}]^{\mathcal{B}}[I] - {}^{\mathcal{B}}[I][\tilde{\boldsymbol{\omega}}]$ is a symmetric matrix with zero diagonal entries, and ${}^{\mathcal{B}}[\dot{I}]$ is a diagonal matrix, we can equate matrix components to solve for the desired differential

Table 1: Satellite Simulation Data

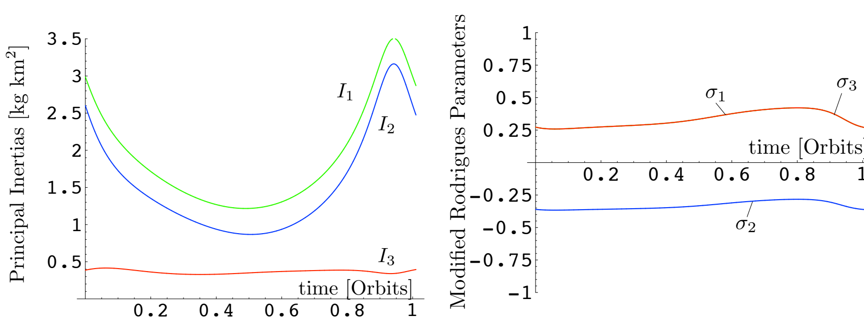
	Sat 1	Sat 2	Sat 3	Sat 4	Sat 5	Units
a	42241.075	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	km
e	0.500	$7{\cdot}10^{-7}$	$9{\cdot}10^{-7}$	$-6{\cdot}10^{-7}$	$-5{\cdot}10^{-7}$	
i	48.000	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	deg
Ω	20.000	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	$0{\cdot}10^{0}$	deg
ω	0.000	$2{\cdot}10^{-5}$	$-1{\cdot}10^{-4}$	$-1{\cdot}10^{-5}$	$-14.5{\cdot}10^{-5}$	deg
M_{t_0}	20.000	$0{\cdot}10^{0}$	$15{\cdot}10^{-5}$	$5{\cdot}10^{-5}$	$18{\cdot}10^{-5}$	deg
m	150.000	50.0	110.0	120.0	140.0	kg

equations.[3,10]

$$\omega_1 = \frac{\xi_{23}}{I_2 - I_3} \qquad \omega_2 = -\frac{\xi_{13}}{I_1 - I_3} \qquad \omega_3 = \frac{\xi_{12}}{I_1 - I_2} \tag{21}$$

$$\dot{I}_1 = \xi_{11} \qquad \dot{I}_2 = \xi_{22} \qquad \dot{I}_3 = \xi_{33} \tag{22}$$

Integrating such differential equations will have well known issues whenever the principal inertias become equal. References 10 and 3 discuss numerical approximations to deal with these. The geometric interpretation of these singularities is that with $I_2 = I_3$, the plane spanned by $\hat{\boldsymbol{b}}_1$ and $\hat{\boldsymbol{b}}_2$ is unique and well defined, but not the individual vectors.



(a) Formation Principal Inertias

(b) Formation Attitude in MRPs

Figure 1: Illustration of the Formation Shape Through Principal Inertias and Formation Attitude through MRPs.

The following numerical simulation illustrates how the formation body frame can be used to describe the overall shape and attitude of a cluster of satellites. The initial conditions of 5 satellites are listed in Table 1. The first satellite is given in terms of osculating orbit elements, while the other satellites are expressed in terms of differences to this first satellite. The full nonlinear inertial equations of motion of each satellite is solved using Eq. (3). No spacecraft charging is active here. Thus, due to all satellites having the same semi-major axis, all relative motions will remain bounded relative to the chief or center of mass motion. Further, since all i and Ω values are equal, there is no out-of-plane relative motion in this cluster. This makes it easier to verify that the computed formation principal inertias I_i and attitude are correct.

Figure 1 shows the resulting formation principal inertias I_i, as well as the $\mathcal{B}$ frame attitude expressed through Modified Rodrigues Parameters (MRPs).[13,17] The $\mathcal{B}$ frame orientation axes are illustrated at 4 different time steps in Figure 2. The axis length is scaled by the principal inertias, to indicate the respective inertia about this axis. Because the relative motion is planar, one principal axis is always pointing perpendicular to the chief orbit plane. Two formation principal axes directions don't vary very much for this case. This is due to the satellites being fairly evenly distributed across the formation. This reinforces the concept that this proposed formation frame does not track individual satellite motions, but only provides information regarding the resulting shape. While the previous discussions regarding relative motion constraints were particular to Coulomb satellite formations, this proposed formation body frame could also be used to describe the shape and orientation of classical formations of satellites.

Conclusion

The angular momentum of the entire formation about an inertial point and the formation center of mass is examined. Because the inertial angular momentum of a Coulomb formation is conserved, the relative motion of the charged spacecraft is subjected to three constraints. If the relative motion is described relative to the formation center of mass, then the center of mass definition provides an additional set of three constraints on the relative motion. Further, the center of mass motion of the formation is studied. Even with Coulomb formations having relative orbit radii of 100 meters, it is shown to be reasonable to treat the center of mass motion as being Keplerian. Finally, a formation coordinate frame is introduced which is similar to that used to track a continuous rigid body motion in space. This formation body frame can be used to describe the overall attitude, shape, size and motion of the formation. Differential equations are provide to integrate the body formation frame.

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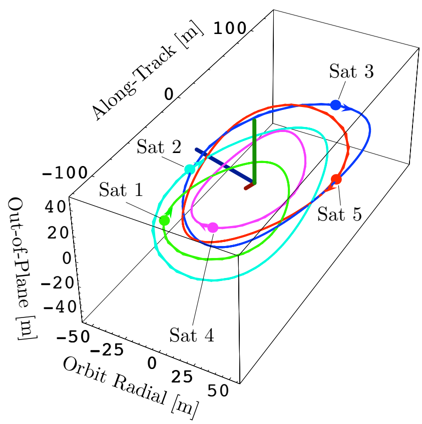
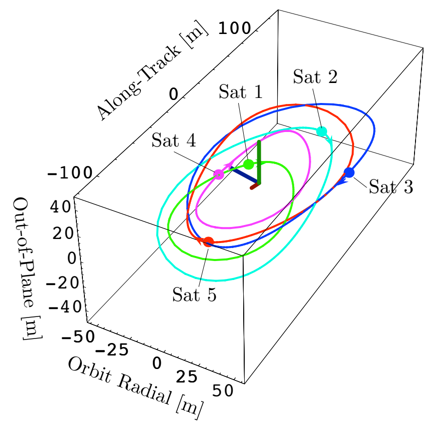
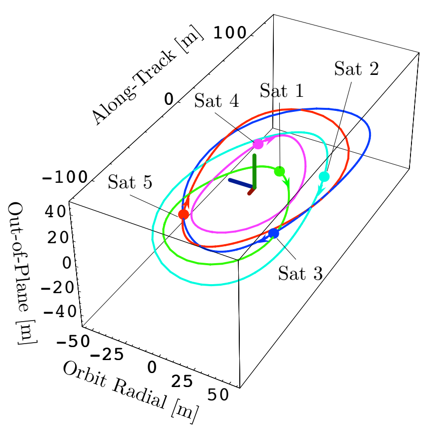


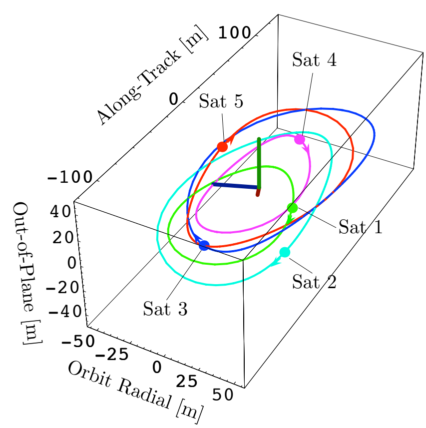
(a) Time Step $t = 0.00$ Periods



(b) Time Step $t = 0.25$ Periods



(c) Time Step $t = 0.50$ Periods



(d) Time Step $t = 0.75$ Periods

Figure 2: Chief Hill frame Illustration of the Formation Body Frame Orientation Axes At Different Time Steps.

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