DYNAMICS AND CONTROL OF MICRO-ROBOT SWARMS: THREE-DIMENSIONAL SURFACE

TECHNICAL REPORT

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The original Micro Robotic Vehicle (MRVs) control problem which allowed MRVs to converge to a point in a plane is expanded to incorporate the motion on a three-dimensional surface. The potential gradient control law is modified to make MRVs avoid steep hills and go around them. Further, a safe guard is built in that does not allow an MRV to tilt beyond a certain angle.

INTRODUCTION

The technical report is a continuation of the original planar MRV study. Here the MRVs are are assumed to be moving on a three-dimensional surface. Line-off-sight issues and gravity effects are not incorporated into this preliminary study. Assuming that the MRVs can sense their tilt, the problem is studied how to make the MRVs avoid steep hills subject to an inequality constraint. Each MRV can only tilt a finite amount before it will tip over. The modified control should clearly keep the MRVs away from such sharp tilts.

THREE-DIMENSIONAL SURFACE GENERATION

To generate a three-dimensional surface over the (x,y)- plane, the following radial basis functions are used.

$$z = \frac{C_{1i}}{1 + \left(x - x_0\right)^2 + \left(y - y_0\right)^2 / C_{2i}} \tag{1}$$

where x_0 and y_0 are the center of the radial basis mountain. Its shape is determined by the two constants C_{1i} and C_{2i}. The parameter C_{1i} determines the height and the

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parameter C_{2i} determines the slope. A surface $S(x, y, z)$ is defined as the finite sum of N radial basis functions.

$$S(x, y, z) = \sum_{i}^{N} \left(z - \frac{C_{1i}}{1 + (x - x_0)^2 + (y - y_0)^2 / C_{2i}} \right) = 0 \tag{2}$$

The gradient of the i-th surface $S_i(x, y, z)$ is a normal vector to the surface at the point (x, y, z). It is defined as

$$\frac{\partial S_i}{\partial x} = \frac{C_{1i}}{\left(1 + (x - x_0)^2 + (y - y_0)^2 / C_{2i}\right)^2} \frac{2}{C_{2i}} (x - x_0) \tag{3}$$

$$\frac{\partial S_i}{\partial y} = \frac{C_{1i}}{\left(1 + (x - x_0)^2 + (y - y_0)^2 / C_{2i}\right)^2} \frac{2}{C_{2i}} (y - y_0) \tag{4}$$

$$\frac{\partial S_i}{\partial z} = 1 \tag{5}$$

Let the actual surface gradient vector components generated by the N radial basis function surfaces be

$$S_x = \sum_{i}^{N} \frac{\partial S_i}{\partial x} \quad S_y = \sum_{i}^{N} \frac{\partial S_i}{\partial y} \quad S_z = \sum_{i}^{N} \frac{\partial S_i}{\partial z} \tag{6}$$

To be used in the potential gradient control law, this surface gradient vector is normalized as the $\boldsymbol{g}$ vector.

$$\boldsymbol{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \frac{1}{\sqrt{S_x^2 + S_y^2 + S_z^2}} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} \tag{7}$$

Let's define the following useful quantity. Let g_{xy} be the vector component of $\boldsymbol{g}$ in the (x, y)-plane.

$$g_{xy} = \sqrt{g_x^2 + g_y^2} \tag{8}$$

Note that if g_{xy} is zero, than the local surface is perfectly level. If g_{xy} approaches 1, then the surface is becoming vertical.

TILT REPULSIVE POTENTIAL

A potential function is sought whose gradient will drive an MRV away from steep slopes and keep it tilted less than some prescribed angle. As before, let the MRV state be given through the planar position (x, y) and the heading angle θ, while the MRV dynamics are defined as

$$\dot{p}_i = B(p_i)\omega_i = \frac{1}{2} \begin{bmatrix} R_r \cos\theta_i & R_l \cos\theta_i \\ R_r \sin\theta_i & R_l \sin\theta_i \\ \frac{R_r}{R_w} & -\frac{R_l}{R_w} \end{bmatrix} \begin{bmatrix} \omega_{r_i} \\ \omega_{l_i} \end{bmatrix} \tag{9}$$

then the control vector $\boldsymbol{\omega}$ is found by the projection of the potential function V_i gradient onto the dynamical system as

$$\omega_i = -\frac{\gamma}{||\nabla V_i||}(B^T B)^{-1} B^T \nabla V_i \tag{10}$$

where ∇V_i is defined as

$$\nabla V_i = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial \theta}\right)^T \tag{11}$$

In order for ∇V_i to provide a corresponding control law, it must depend on x, y and θ. If not, then ∇V_i would be zero and have no influence on the control law. However, the local surface gradient is not directly related to the MRV state relative to the target state.

Let the vector $\boldsymbol{g} = (g_x, g_y)^T$ be the projection of the local surface unit gradient vector of the i-th MRV position (x_i, y_i) onto the horizontal x-y plane. It is immediately clear that for the MRV to avoid climbing up the hill, it would have to move in the $\boldsymbol{g}$ direction. To drive it in that direction, a repulsive potential is placed at the location (x_m, y_m) given by

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \kappa \boldsymbol{g} \tag{12}$$

where κ is a yet unknown parameter. However, if the local slope is very steep, then the center of the repulsive potential should be located very close to (x_i, y_i) which leads to κ being very small. If the local slope is very small, then κ should be very large to place the center of the repulsive potential far from the current MRV location. Since the MRVs are only allowed to tilt a finite amount φ_{max}, the κ term should tend to zero as g_{xy} approaches the maximum allowable g_{max}. The term g_{max} is related to φ_{max} through

$$g_{max} = \sin \varphi_{max} \tag{13}$$

Therefore κ is defined as

$$\kappa = \left(\frac{g_{max}}{g_{xy}}\right)^2 - 1 \tag{14}$$

As $g_{xy} \to g_{max}$, then $\kappa \to 0$ as desired.

The relative distances to this repulsive potential are

$$\Delta x = x_m - x_i \quad \Delta y = y_m - y_i \tag{15}$$

which can be rewritten using Eq. (12) to

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = -\kappa \boldsymbol{g} \tag{16}$$

The tilt repulsive potential V^{tr} is then defined as

$$V^{tr} = \frac{1}{2}\frac{k_5}{r^2} + \frac{1}{2}\beta^2 \tag{17}$$

where the term r^2 is defined as

$$r^2 = \Delta x^2 + \Delta y^2 = \kappa^2 \boldsymbol{g}_{xy}^2 \tag{18}$$

and the angle β is defined the same as in the planar Bug Control problem as

$$\beta = \phi - \theta - \pi \tag{19}$$

Including this angle will drive the MRVs to point down the slope. Note that including this heading information is very important in this potential gradient control law. The MRVs cannot move sideways. Therefore, in order to move on a new heading, they have to rotate by having uneven track speeds. Without the heading information the MRV would never try to rotate. It would simply drive in a straight line until it has found a local minimum of the surrounding potential field.

The gradient of V^{tr} is then given by

$$\nabla V^{tr}(x_i, y_i, \theta_i) = \begin{pmatrix} k_5 \frac{\Delta x}{r^4} + k_6 \beta \frac{\Delta y}{r^2} \\ k_5 \frac{\Delta y}{r^4} - k_6 \beta \frac{\Delta x}{r^2} \\ -k_6 \beta \end{pmatrix} \tag{20}$$

NUMERICAL EXAMPLE

To illustrate the presented potential gradient control law the following simulation was run. The code for the planar MRV simulation was modified to incorporate information about a three-dimensional surface. One mountain was placed at (-2,-8) with c_1=1 and $c_2 = 5$. The potential gains are set to $k_5 = 10$ and $k_6 = 6$. The maximum allowable tilt angle is 45 degrees. The initial MRV position is at $(-7, -17)$. The resulting motion is shown in Figure 1. The MRV first reorients itself to face off with the target at $(0, 0)$ and then moves towards it. Without V^{tr} present, the MRV would roll right over the peak. With V^{tr} the MRV starts to veer to the right and rolls around the mountain. Note that this is a very simple illustration only. This "mountain" is more like a small bump along the way, therefore the MRV is not too concerned about rolling over parts of it. How strongly the MRVs are repelled from this "mountains" can be controlled with the gains k_5 and k_6 along with the maximum tilt factor g_{max}.

C-CODE OVERVIEW

This iteration of the code is called **grad9.c**. It is a straight continuation of **grad8.c** with an extra tilt avoidance potential function added on. The simulation parameters are read in from the support file called *data.3d*. The three-dimensional surface is defined trough the data file *mountains*. This file contains information on how many radial function peaks are present, what the maximim allowable g_{max} is and that the tilt repulsive potential gains k_5 and k_6 are. The code then reads in the (x, y) location and the c_1 and c_2 parameters for each radial basis functions.

After calculating the potential gradients due to the target potential and due to repelling from other MRVs, the addition potential gradient is calculated to avoid exessive tilting.

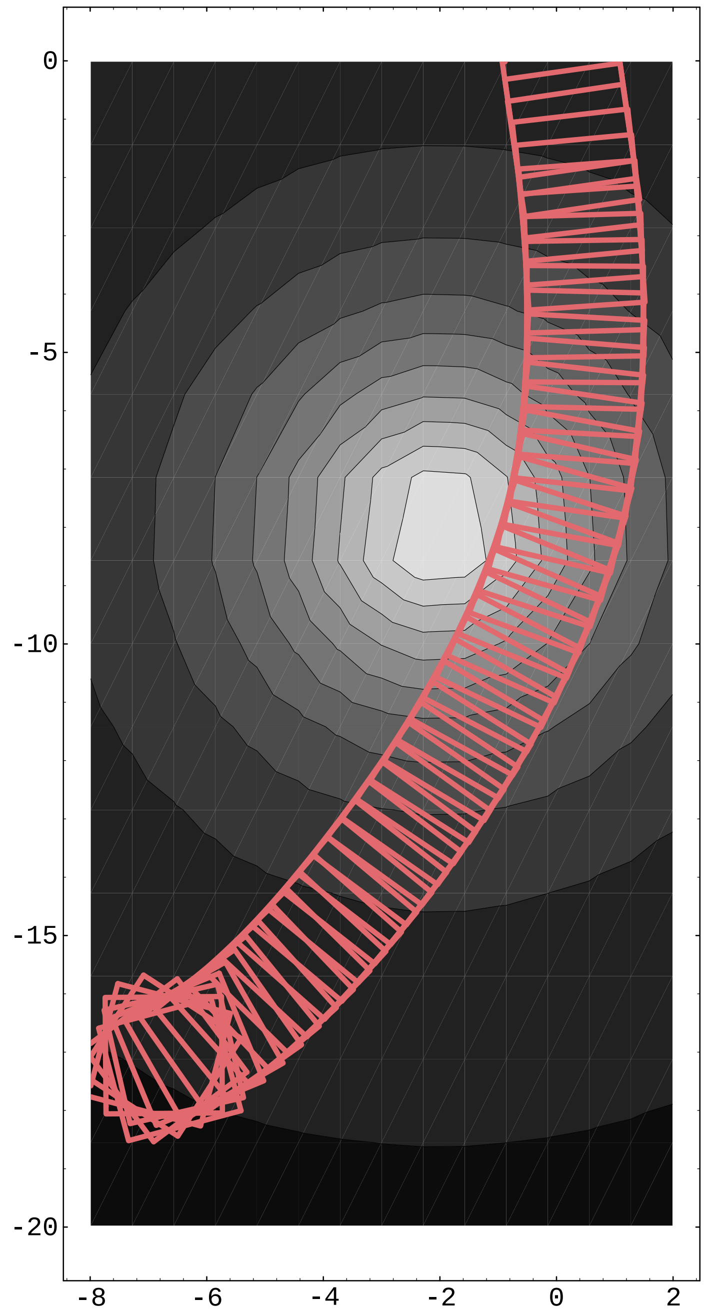

Figure 1 Illustration of MRV Mountain Avoidance