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ONE-DIMENSIONAL 3-CRAFT COULOMB STRUCTURE CONTROL

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Abstract

A Coulomb structure is a cluster of satellites that can maintain or change its shape through inter-vehicle electrostatic forces generated by on-board charge control devices. This paper investigates the charged dynamics of a linear 3-craft cluster. The 1D-restricted control of a three-body coulomb structure is developed. This control can be used to stabilize the motion of charged spheres on a 1D hover test track. A feedback control law based on Lyapunov stability analysis is developed to control the relative distances between the satellites. Exploiting the null space of the input-output relationship, a minimal-charge routine is developed guaranteeing that the final control strategy is implementable with real charges at all times. Numerical simulations illustrate the performance, and also demonstrate that by slightly releasing the criteria for the search routine to switch between two eligible intervals, the system can avoid chatter in the charge time histories, and thus can reduce the power expense and the burden on the hardware.

Introduction

King et al. [1] originally discussed the novel method of exploiting Coulomb forces for formation flying in 2002. Since then, many papers have been published in this area. Coulomb forces are used to control a tight formation up to 100 meters. Electrostatic force fields are generated to control the formation's shape and size. There are some other promising techniques in close formation flying such as the Electric Propulsion(EP) [1] and Electro-Magnetic Formation Flying(EMFF) [2]. EP systems generate forces by expelling ionic plumes. The ionic plumes can disturb the motions of nearby spacecraft, and the intensive and caustic charge plumes are also threatening to sensitive instruments. The EMFF method controls relative separation and attitude of the formation by creating electromagnetic dipoles on each spacecraft in concert with reaction wheels. In contrast to EP method, the Coulomb formation flying technique has no exhausting plume contamination issues. The Coulomb force field is also simpler to model than electromagnetic force field, and the strength does not drop off as fast as electromagnetic force field. Coulomb force is also practically promising in that the required forces can be created with milliwatts of power and can be controlled on a millisecond time scale [3]. In addition, Coulomb force control is 3-5 orders of magnitude more fuel-efficient than EP [1]. This is an essential advantage in long-term space missions.

Many practical problems in Coulomb formation flying have been investigated. Hyunsik Joe et al. introduced a formation coordinate frame which tracked the principal axes of the formation in [4]. Gordon G. Parker et al. presented a sequential control strategy for arranging N charged bodies into an arbitary geometry using $N+3$ participating bodies in [5]. The paper overcame two challenging problems of Coulomb force control: the Coulomb force coupling and unimplementable control solutions arising from the square force nonlinearity. First order differential orbit element constraints for Coulomb formation are studied in [6]. Arun Natarajan et al. developed a charge feedback law to stabilize the relative distance between two satellites of a Coulomb tether formation in [7]. By exploiting the gravity gradient torque, the attitude of the Coulomb tether formation can also be stabilized.

This paper discusses another application of Coulomb force formation, Coulomb virtual structures. A Coulomb structure is a virtual structure composed of several spacecraft. It controls its shape and size by utilizing the inter-spacecraft electrostatic forces. This virtual structure control can be used in large scale distributed spacecraft concepts. As a fundamental study of a simple Coulomb structure, the paper considers a 1D Coulomb structure consisting of three spacecraft. Based on Lyapunov stability analysis, a control law is introduced to generate the charge products to stabilize the separation distances. As a general problem in Coulomb formation flying, the charge products are not guaranteed to produce real charges at all times. Sometimes imaginary values of charges are obtained from the basic control law. An optimal search routine is of interest to reach a solution that not only guarantees implementability, but also can minimize the spacecraft charges in stabilizing the formation to a rigid virtual structure. Numerical simulations show the resulting performance of the 1D 3-craft Coulomb structure control system.

Coulomb structure dynamics

Let the Coulomb structure consist of 3 bodies with masses m_i, and they are restricted to move in one-dimension only. The inertial positions of the three bodies are given through their inertial coordinates x_i. The charges q_i always appear in pairs q_iq_j both in the dynamic functions and in the control formulation. Charge products are introduced as

$$Q_{ij} = q_i q_j \tag{1}$$

This approach quickly leads to the problem of physical feasibility in extracting individual charges q_i which is addressed in the later sections. Without loss of generality, assume that $x_1 < x_2 < x_3$. The equations of motion of the charged bodies are given through

$$m_1\ddot{x}_1 = k_c\left[-\frac{Q_{12}}{(x_2-x_1)^2}-\frac{Q_{13}}{(x_3-x_1)^2}\right] \tag{2}$$

$$m_2\ddot{x}_2 = k_c\left[\frac{Q_{12}}{(x_2-x_1)^2}-\frac{Q_{23}}{(x_3-x_2)^2}\right] \tag{3}$$

$$m_3\ddot{x}_3 = k_c\left[\frac{Q_{13}}{(x_3-x_1)^2}+\frac{Q_{23}}{(x_3-x_2)^2}\right] \tag{4}$$

where $k_c = 8.99\times 10^9 \mathrm{C}^{-2}\cdot\mathrm{N}\cdot\mathrm{m}^2$ is the Coulomb's constant. A charge feedback law is required to control the relative motion of the three-body Coulomb structure and make the formation assume a specific shape.

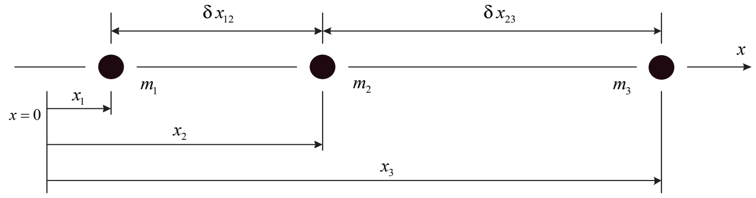


Figure 1: Illustration of positions and coordinates of the 3-body system.

Control strategy

Not all of the inertial x_i states can be controlled independently. Because the spacecraft charges produce formation internal forces, the momentum of the Coulomb cluster must be conserved if there are no other external forces acting on it. As a result it is not possible to completely control the three inertial coordinates x_i. For the 1D motion considered in this paper, the momentum conservation imposes one constraint on the system. Thus, the motion of the three-body system only has two degrees of freedom. To control the shape of the 1D-restricted 3-craft Coulomb structure, it is equivalent to control the two relative distances:

$$\delta x_{12} = x_2 - x_1, \qquad \delta x_{23} = x_3 - x_2 \tag{5}$$

Here the third distance δx_{13} is determined by δx_{12} and δx_{23}. The goal of the feedback control law is to drive $[\delta x_{12}, \delta x_{23}]^T$ to desired values $[\delta x^*_{12}, \delta x^*_{23}]^T$ that yield a specific virtual structure shape. Note that these two relative position coordinates are independent coordinates. The holonomic constraint due to momentum conservation has already been incorporated into the relative motion coordinate choice.

Control law based on Lyaponov stability

For notational convenience the vector $\boldsymbol{\xi} = [a,b,c]^T$ is introduced and defined as

$$a = \frac{k_c Q_{12}}{(x_2 - x_1)^2}, \quad b = \frac{k_c Q_{23}}{(x_3 - x_2)^2}, \quad c = \frac{k_c Q_{13}}{(x_3 - x_1)^2} \tag{6}$$

Using Equation 2− 4 and 6, the equations of motion of the relative position coordinates are given by

$$\delta\ddot{x}_{12} = \ddot{x}_2 - \ddot{x}_1 = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) a - \frac{1}{m_2} b + \frac{1}{m_1} c \tag{7}$$

$$\delta\ddot{x}_{23} = \ddot{x}_3 - \ddot{x}_2 = -\frac{1}{m_2} a + \left(\frac{1}{m_2} + \frac{1}{m_3}\right) b + \frac{1}{m_3} c \tag{8}$$

Define the state vector $\mathbf{X}$ to be the relative motion tracking error

$$\mathbf{X} = \begin{bmatrix} \Delta x_{12} \\ \Delta x_{23} \end{bmatrix} = \begin{bmatrix} \delta x_{12} - \delta x_{12}^* \\ \delta x_{23} - \delta x_{23}^* \end{bmatrix} \tag{9}$$

Assuming that the desired relative position coordinates are constants, the tracking error dynamics is expressed using $\mathbf{X}$ as

$$\ddot{\mathbf{X}} = \underbrace{\begin{bmatrix} \frac{1}{m_1} + \frac{1}{m_2} & -\frac{1}{m_2} & \frac{1}{m_1} \\ -\frac{1}{m_2} & \frac{1}{m_2} + \frac{1}{m_3} & \frac{1}{m_3} \end{bmatrix}}_{[A]} \boldsymbol{\xi} \tag{10}$$

Define the following Lyapunov function in terms of $\mathbf{X}$:

$$V(\mathbf{X}) = \frac{1}{2}\mathbf{X}^T [K]\mathbf{X} + \frac{1}{2}\dot{\mathbf{X}}^T\dot{\mathbf{X}} \tag{11}$$

where $[K]$ is a symmetric, positive definite gain matrix. Next, the derivative of V is set to be equal to a negative definite function

$$\dot{V} = \dot{\mathbf{X}}^T [K]\mathbf{X} + \dot{\mathbf{X}}^T\ddot{\mathbf{X}} = -\dot{\mathbf{X}}^T [P]\dot{\mathbf{X}} \tag{12}$$

where $[P]$ is a symmetric, positive definite velocity feedback gain matrix. Thus, the stabilizing control law must satisfy

$$[A]\boldsymbol{\xi} + [K]\mathbf{X} + [P]\dot{\mathbf{X}} = 0 \tag{13}$$

Note that $[A]$ is a 2×3 real-valued matrix, so there is an infinity of solutions of $\boldsymbol{\xi}$ that satisfy this equation. Here the minimum norm inverse is chosen (which will minimize the norm of the $\boldsymbol{\xi}$ vector), and yields

$$\hat{\boldsymbol{\xi}} = [A]^T \left([A][A]^T\right)^{-1} \left(-[K]\mathbf{X} - [P]\dot{\mathbf{X}}\right) \tag{14}$$

Note that $\hat{\boldsymbol{\xi}}$ is not the only solution to $\boldsymbol{\xi}$, and it doesn't minimize the spacecraft charges q_i, but rather the $\boldsymbol{\xi}$ vector. Substituting Equation 6 into Equation 10, yields

$$\ddot{\mathbf{X}} + [P]\dot{\mathbf{X}} + [K]\mathbf{X} = 0 \tag{15}$$

Equation 15 shows that the closed-loop dynamics is in a linear form with proportional position and rate feedback.

From the definition of $\boldsymbol{\xi}$ in Equation 6, individual q_i values can be calculated through

$$q_1 = \sqrt{\frac{ac}{bk_c}} \frac{|\delta x_{12}||\delta x_{13}|}{|\delta x_{23}|} \tag{16}$$

$$q_2 = \text{sign}(bc)\sqrt{\frac{ab}{ck_c}} \frac{|\delta x_{12}||\delta x_{23}|}{|\delta x_{13}|} \tag{17}$$

$$q_3 = \text{sign}(c)\sqrt{\frac{bc}{ak_c}} \frac{|\delta x_{23}||\delta x_{13}|}{|\delta x_{12}|} \tag{18}$$

Notice that the singularity problem occurs when some elements of $\hat{\boldsymbol{\xi}}$ are equal to zero. Because $\hat{\boldsymbol{\xi}}$ is physically determined by charges, there are two possible cases for elements of $\hat{\boldsymbol{\xi}}$ to be equal to zero. The first is that two elements of $\hat{\boldsymbol{\xi}}$ equal zero, the other is $\hat{\boldsymbol{\xi}} = 0$. The first case can be avoided by performing a search routine in the null space of $[A]$ matrix which will be discussed in the following sections. The second case indicates that $q_1 = q_2 = q_3 = 0$, this state occurs only either when $\Delta\mathbf{x} = \mathbf{0}$ and $\Delta\dot{\mathbf{x}} = \mathbf{0}$, which means the system has reached the desired state, or due to $(-[K]\mathbf{X} - [P]\dot{\mathbf{X}})$ being zero temporally.

Now consider general cases where $a \cdot b \cdot c \neq 0$. Note that $a \cdot b \cdot c < 0$ yields imaginary values of q_i. But charges must always be real numbers, so $a \cdot b \cdot c < 0$ is not an implementable solution. This is a fundamental issue with developing any charge feedback law. If the constrained inertial positions x_i are controlled instead of the unconstrained relative position coordinates δx_{ij}, this approach would be at a road block. However, note that the particular $\hat{\boldsymbol{\xi}}$ value is obtained by looking for a minimum norm solutions to this vector in Equation 14. However, there is an infinity of solutions that satisfy Equation 13 Using the null space of $[A]$, all possible $\boldsymbol{\xi}$ values are parameterized as

$$\boldsymbol{\xi} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \tag{19}$$

The control problem is rewritten to find a parameter γ that satisfies

$$f(\gamma) = a \cdot b \cdot c = (\hat{a} - \gamma)(\hat{b} - \gamma)(\hat{c} + \gamma) > 0 \tag{20}$$

This inequality constraint guarantees that the charges q_i are real, and also ensures that the temporal singularity case $a \cdot b \cdot c = 0$ does not occur.

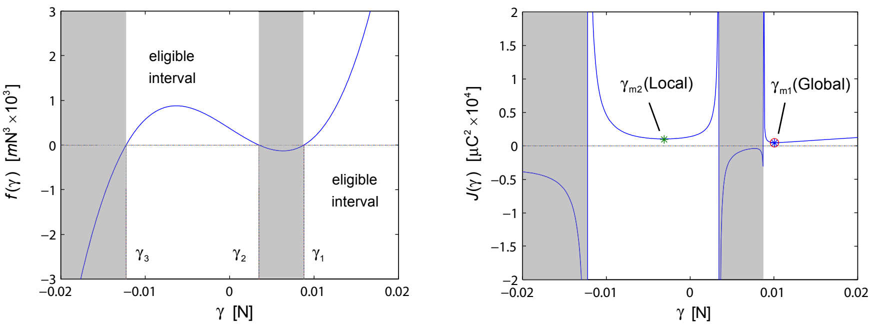
Optimal search routine

Any choice of γ in Equation 19 provides the required stabilizing charging behavior, but sometimes the solution is not implementable. A parameter γ that satisfies the inequality in Equation 20 makes the solution physically implementable with real charge q_i solutions. In fact, note that the null space of the input matrix $[A]$ can be used to charge up the vehicles and not cause any relative motion to occur. The $\hat{\boldsymbol{\xi}}$ vector is found such that the norm of the vector $\boldsymbol{\xi}$ is minimized. However, this doesn't correspond to a more logical optimal solution where the spacecraft charges q_i are minimized. Define a charge cost function $J(\gamma)$ as

$$J(\gamma) = \sum_{i=1}^{3} q_i^2 \tag{21}$$

The optimal solution $\boldsymbol{\xi}$ that minimizes spacecraft charges q_i corresponds to the optimal γ_m that satisfies the inequality constraint in Equation 20, and at the same time minimizes the charge cost function $J(\gamma)$.

Consider the constraint inequality in Equation 20, where $(\hat{a}, \hat{b}, \hat{c})$ are given by Equation 14. There are three real roots for the equation $f(\gamma) = 0$, and the roots are just $(\hat{a}, \hat{b}, -\hat{c})$. Rearrange the roots in descent order and denote as $(\gamma_1, \gamma_2, \gamma_3)$, where $\gamma_1 \geq \gamma_2 \geq \gamma_3$. The solution to the constraint in Equation 20 turns out to be $\gamma > \gamma_1$ or $\gamma_3 < \gamma < \gamma_2$. If $\gamma_2 = \gamma_3$, then the solution is simply $\gamma > \gamma_1$. Figure 2(a) shows a numerical example of $f(\gamma)$ and $(\gamma_1, \gamma_2, \gamma_3)$.



(a) $f(\gamma)$ and $(\gamma_1, \gamma_2, \gamma_3)$. (b) γ_m search result.

Figure 2: A sample shows γ_m search.

Thus the optimization problem can be formulated as follows: search the optimal γ_m of the charge cost function $J(\gamma)$ within two open intervals (γ_1, ∞) and (γ_3, γ_2). The search routine using secant method is shown in Figure 3.

Once γ_m is obtained, an implementable solution that minimizes the norm of the charge vector (q_1, q_2, q_3) is also reached. Figure 2(b) shows an example of the search result at one instant, where γ_{m1} and γ_{m2} are two local optimal points.

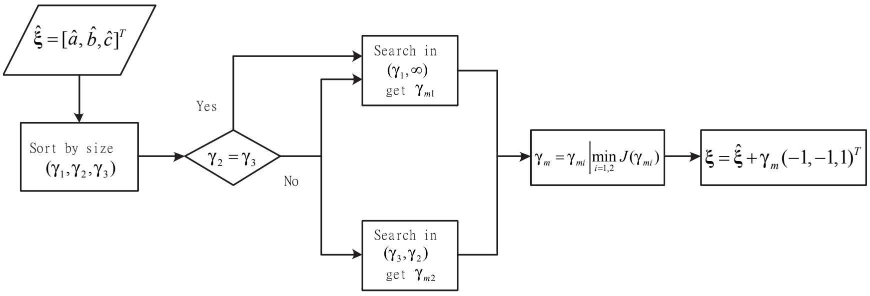


Figure 3: Illustration of γ_m search routine.

Notice that generally there are two eligible intervals in the search routine. Sometimes this may introduce chatter because γ_m switchs between γ_{m1} and γ_{m2} when $J(\gamma_{m1})$ and $J(\gamma_{m2})$ are very close. To reduce the chatter of the charge history, one approach is to change the criteria for γ_m to switch between the two intervals. If $\gamma_m(i) = \gamma_{m1}(i)$, then $\gamma_m(i+1) = \gamma_{m2}(i+1)$ if and only if $J(\gamma_{m2}) < \alpha \cdot J(\gamma_{m1})$, where $0 < \alpha \leq 1$. Following numerical simulations will show results of this trade off.

Numerical simulations

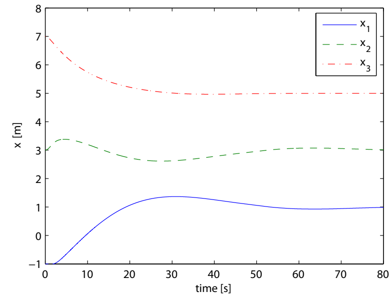
The charging control law is determined by first computing $\hat{\boldsymbol{\xi}}$ using Equation 6, then searching the optimal γ_m to calculate (a,b,c). Individual charges q_i are determined using Equations 16−18. The solution is physically implementable at all times and has the minimal charge cost at each step when $\alpha = 1$. Figure 4 shows the simulation results with parameters $m_1 = m_2 = m_3 = 1$kg, $\alpha = 1$, $\delta x_{12}^* = 2$m, $\delta x_{23}^* = 2$m, and initial states $\mathbf{X}(0) = [-1,3,7]^T$m, $\dot{\mathbf{X}}(0) = \mathbf{0}$m/s. The gain matrices of the controller are

$$[K] = 0.01 \cdot \mathrm{diag}(1,1), \qquad [P] = 0.12 \cdot \mathrm{diag}(1,1) \tag{22}$$

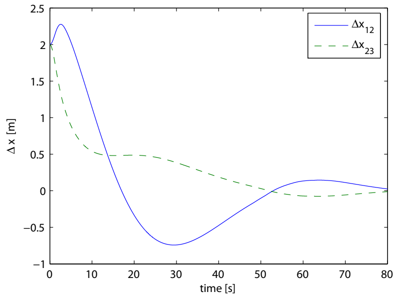
Figure 5 shows simulation results under the same conditions as in Figure 4 except that $\alpha = 0.7$. It can be seen that comparing with the charge history in Figure 4 the chatter effect has been greatly reduced in Figure 5.

Conclusion

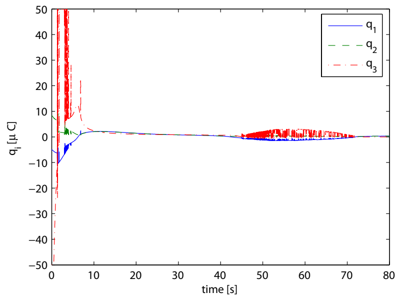
A three-body 1D Coulomb structure control strategy is presented in this paper. The purpose of the controller is to maintain a fixed shape of the 1D Coulomb structure by controlling the distances of the charged particles. Lyaponov stability analysis is used in developing the basic control law to derive charge products. But real charges cannot always be extracted from these charge products. A search routine in the null space of



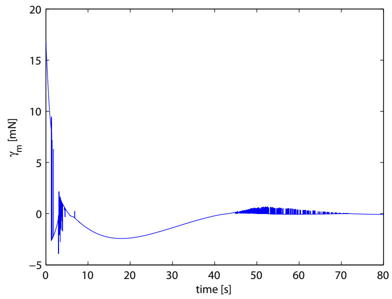
(a) inertial position



(b) relative position tracking error

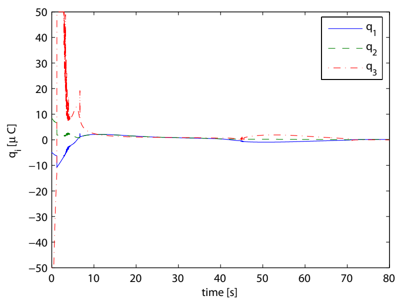


(c) spacecraft charges

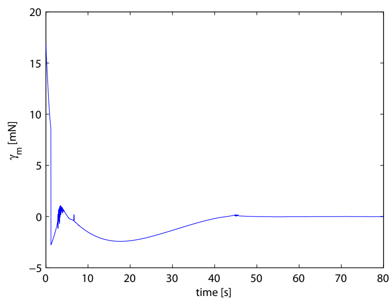


(d) parameter γ_m

Figure 4: Illustration of the control effect with $\alpha = 1$.



(a) spacecraft charges



(b) parameter γ_m

Figure 5: Illustration of the control effect with $\alpha = 0.7$.

the input matrix is designed to find the optimal solution that minimizes the norm of the charge vector and makes the solution implementable. Chatter may arise because of the high-frequency switching of the optimal point between two eligible intervals. Simulations show that this phenomenon can be reduced by softerning the criteria for the switching.

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