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Nonlinear Control of an Inverted Pendulum using Two Reaction Wheels

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Introduction and Motivation

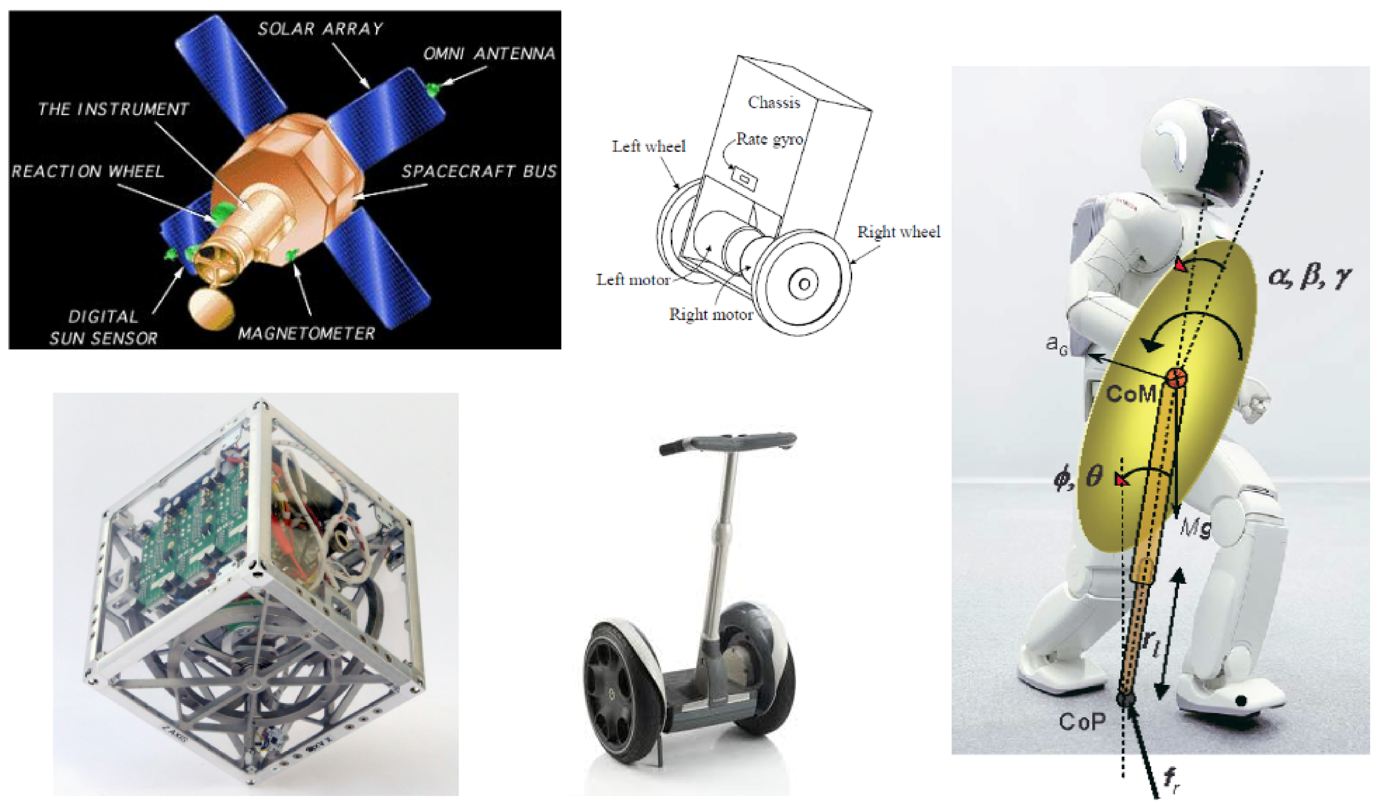


Figure 1: Examples of attitude control

Objective

To control an inverted pendulum with two reaction wheels using a nonlinear controller that has the feature of choosing how active each control action is.

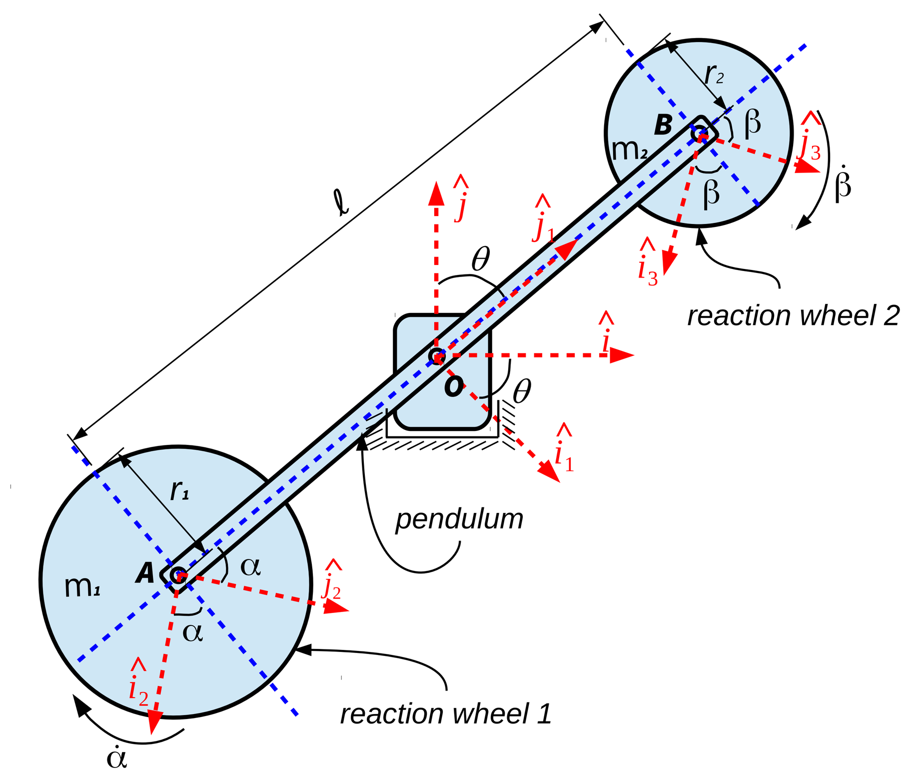
Model of the Two Reaction Wheels Pendulum

The angular momentum of the pendulum is given by:

$$ {}^B\boldsymbol{H} = \left(I^O_{zeq}\dot{\theta} + I^O_{zw1}\dot{\alpha} + I^O_{zw2}\dot{\beta}\right)\hat{k}_1 \quad (1)$$

The rate of the angular momentum, ${}^B\dot{\boldsymbol{H}}$ is equal to the torque sum relative to the fixed point O:

$$ {}^B\dot{\boldsymbol{H}} = \frac{{}^Bd}{dt}(\boldsymbol{H}) + \underbrace{{}^B\boldsymbol{\Omega} \times {}^B\boldsymbol{H}}_{=0} \equiv {}^B\boldsymbol{T}_O \quad (2)$$



The equation of motion and the motor torque equations for 2-RWP are:

$$I^O_{zeq}\ddot{\theta} + I^O_{zw1}\ddot{\alpha} + I^O_{zw2}\ddot{\beta} = (m_1 - m_2)g\frac{\ell}{2}\sin\theta \quad (3)$$

$$I^A_{zw1}\ddot{\theta} + I^A_{zw1}\ddot{\alpha} = T_1 \quad (4)$$

$$I^B_{zw2}\ddot{\theta} + I^B_{zw2}\ddot{\beta} = T_2 \quad (5)$$

Design of the Controller

It is necessary to select a candidate to be a Lyapunov function:

$$\mathcal{V}(\delta\theta, \delta\dot{\theta}) = \frac{\delta\dot{\theta}^2}{2} + K\frac{\delta\theta^2}{2} \quad (6)$$

where $\delta\dot{\theta} = \dot{\theta} - \dot{\theta}_r$ and $\delta\theta = \theta - \theta_r$. The derivative of the candidate for Lyapunov function is set to be equal to a negative definite function:

$$\dot{\mathcal{V}}(\delta\theta, \delta\dot{\theta}) = \delta\dot{\theta}\left(\delta\ddot{\theta} + K\delta\theta\right) = -P\delta\dot{\theta}^2 \quad (7)$$

The closed-loop dynamics is:

$$\ddot{\theta} - \ddot{\theta}_r + K\delta\theta + P\delta\dot{\theta} = 0 \quad (8)$$

To prove that the proposed control is asymptotically stabilizing, the higher order derivatives of the candidate for the Lyapunov function must be analyzed.

$$\ddot{\mathcal{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2P\delta\dot{\theta}\delta\ddot{\theta} = 0 \quad (9)$$

The third order derivative is given by:

$$\dddot{\mathcal{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2P\delta\ddot{\theta}^2 \quad (10)$$

and from the closed-loop dynamics

$$\dddot{\mathcal{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2PK^2\delta\theta^2 < 0 \quad (11)$$

The control law is:

$$\underbrace{\begin{bmatrix} I^O_{zw1} & I^O_{zw2} \\ I^O_{zeq} & I^O_{zeq} \end{bmatrix}}_{[Q]} \begin{Bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{Bmatrix} = \underbrace{\frac{m_1 - m_2}{I_{eq}} g\frac{\ell}{2}\sin\theta - \ddot{\theta}_r + K\delta\theta + P\delta\dot{\theta}}_{\boldsymbol{L}_r} \quad (12)$$

$$\boldsymbol{\eta} = \begin{Bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{Bmatrix} = [W][Q]^T\left([Q][W][Q]^T\right)^{-1}\boldsymbol{L}_r \quad (13)$$

Results

The results present three different situations. First, the control of the 2-RWP is performed using both control actions. Afterward, each reaction wheel mode is switched off, one at a time, to evaluate the control of the 2-RWP using only one actuator instead of two.

Figure 1: Control of the Two reaction wheels pendulum in the inverted position ($\theta = 0°$). — indicates the desired rate.

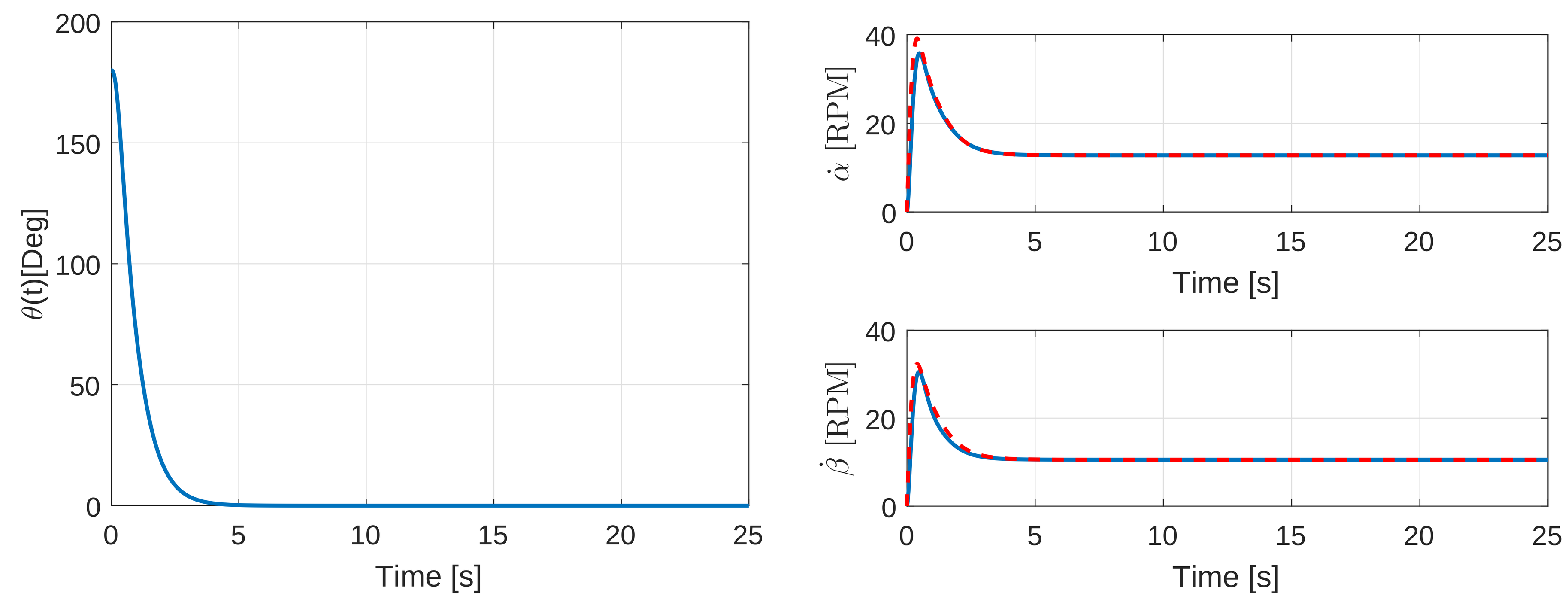


Figure 2: Control of the Two reaction wheels pendulum in the inverted position ($\theta = 0°$) using only reaction wheel 1 mode. — indicates the desired rate.

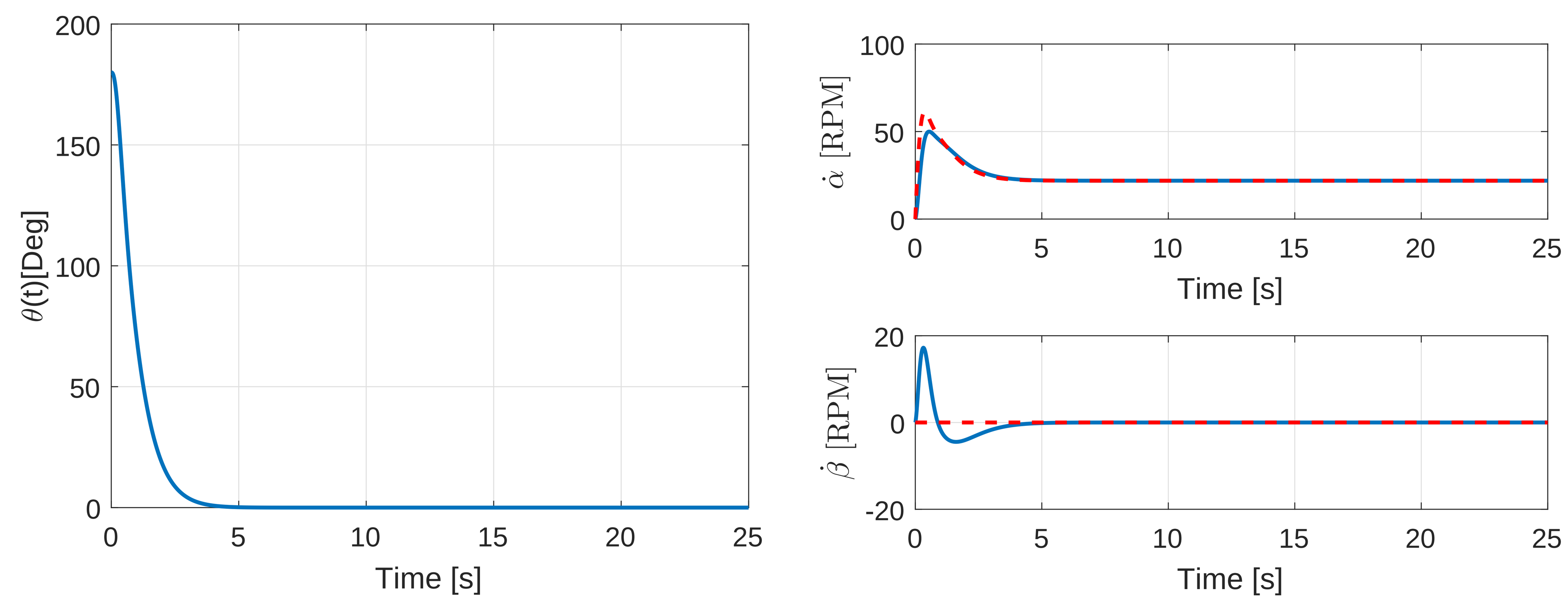
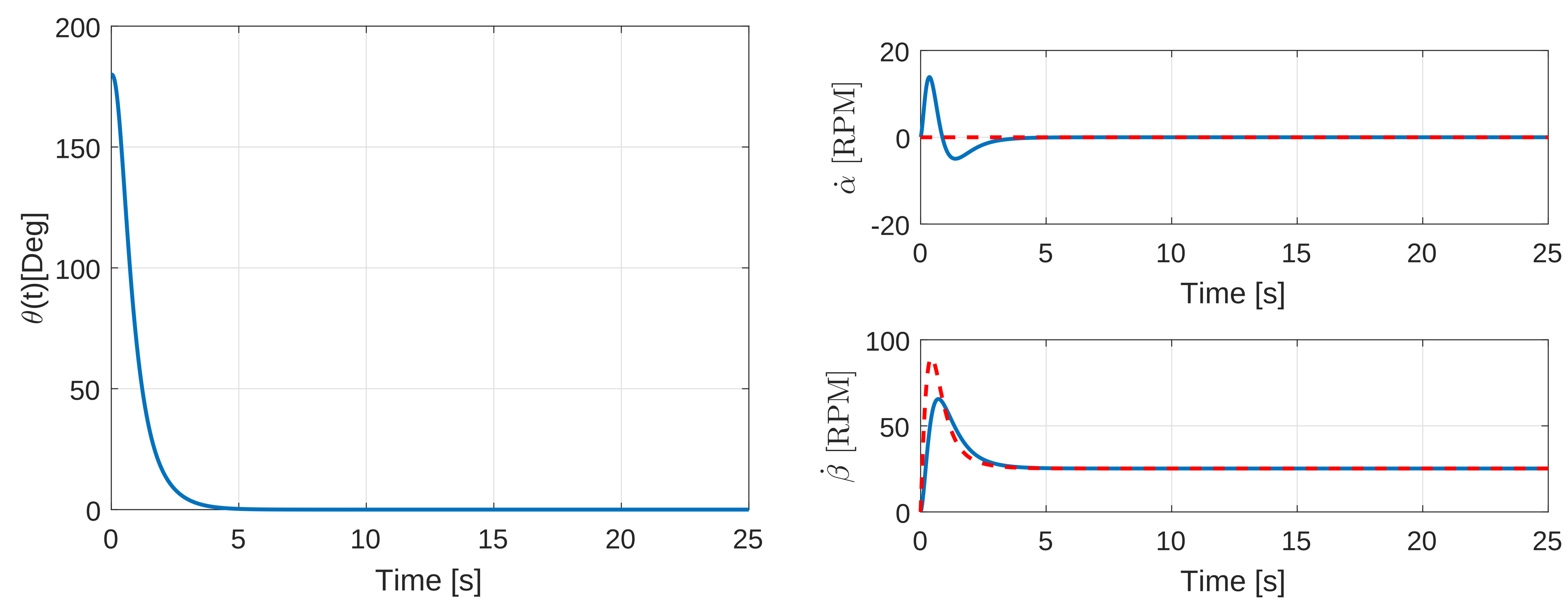


Figure 3: Control of the Two reaction wheels pendulum in the inverted position ($\theta = 0°$) using only reaction wheel 2 mode. — indicates the desired rate.



Final Remarks

This work presented a non-usual pendulum configuration that is actuated by two reaction wheels. Also, a nonlinear controller was derived from Lyapunov control theory. The stability of the control law proposed was proved. The controller proposed also has the feature of choosing how active each control mode is. The results have shown that the 2-RWP can be controlled using both reaction wheels or using just one.

Acknowledgments

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