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SATELLITE FORMATION FLYING: ROBUST ALGORITHMS FOR PROPULSION, PATH PLANNING AND CONTROL

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Abstract—**Propulsion, path planning and control of satellite formations in Geostationary Earth Orbits (GEO) and other high Earth Orbits is a challenging problem. This paper presents the results of the analysis of two types of controllers for satellite formation flying; the first one linear, using classical Proportional-Derivative (PD) control, and the second one nonlinear, using Sliding Mode Control (SMC). The Artificial Potential Field (APF) method is used for collision-free path planning of the satellites in the formation. The satellites are propelled using Coulomb forces and conventional electric/ion thrusters. This hybrid propulsion is more efficient as it minimizes the use of on-board power. Simulation results show that for the formation flying scenario considered in this study, the sliding mode controller gives better performance over the PD controller. Simulation results prove that for the tetrahedron formation considered in this study, both the control effort and drift in the geometric center of the formation are less when a sliding mode controller is used.**

Keywords— **Formation flying; Path planning; Artificial potential field; Sliding mode control.**

I. Introduction

THE concept of a formation which allows multiple geostationary spacecraft to share a common orbital slot is introduced in [1]. The main advantage of spacecraft formation flying is that the functionality of one big spacecraft can be distributed among several smaller spacecraft working in co-operation, thereby reducing the total weight and launch costs. Moreover, upgrades or repairs could be performed by replacing any obsolete or disabled spacecraft in the formation [2]. In recent years, there has been much interest in close-proximity (10-150m) satellite formation flying. A novel satellite formation flying concept exploiting the inter-satellite electrostatic force can be found in [3] and [4]. By generating different charges on satellites in close proximity, each craft exerts a force on all the other satellites. This force can potentially be exploited to control the relative motion of the satellites [5]. Electrostatic forces are used to control the relative motion of a satellite formation in [6]. In [7], an algorithm is developed to determine the steady state equilibrium in which the sum of acceleration on each satellite in the formation is zero. With Coulomb control, all forces are internal and so the Coulomb forces cannot alter the total inertial linear and rotational momentum [8], [9]. Use of Coulomb forces can allow the relative motion of satellites to be controlled without any contaminations [10]. Coulomb thrusting makes use of a renewable source of electrical energy and is essentially free from contaminations due to its extremely high fuel efficiency. References [11] and [12] estimate that Coulomb forces of the order of 10-1000 micro-Newtons, comparable to the thrust developed by conventional electric propulsion, can be produced on short timescales, using less than 1 Watt of on board power.

A challenging problem with spacecraft formation flying is that the spacecraft must be autonomous and able to generate and correct their own relative positions with limited guidance from the ground. The first work in this field, as in the relative position case, stems from the work done on automatic rendezvous and docking control of two spacecraft, as was done on the Gemini/Apollo missions. Later it was utilized in the Space Shuttle, Skylab and Soyuz space stations [13]. Spacecraft position and attitude control, collision-free optimal trajectory generation and disturbance rejection needs to be performed in optimal time and consuming minimum fuel. In [14], a control technique to rotate the entire formation about a given axis and synchronize the individual spacecraft with the formation is proposed. Both position and attitude are controlled, and the error is proven to decay to zero exponentially, though under the assumption of no environmental disturbances and implementation difficulties. A general optimization based control methodology to solve constrained trajectory generation problems for station keeping and reconfiguration of fully actuated low thrust micro-satellites is developed in [15]. An off-board computed procedure using the theory of optimal control for the design of formation reconfiguration is presented in [16]. Although these results are encouraging, much work is needed before this technology is feasible for actual space applications.

Formation flying requires an intelligent path planner and robust control system in order that spacecraft can avoid collisions and reach their target locations. A well-known approach to collision-free path planning of terrestrial robots is using the Artificial Potential Field (APF) method developed in [17]. The APF method is rapidly gaining

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popularity in many practical applications, as it is computationally less expensive than the global approach and provides a simple and effective path planner for obstacle avoidance. It has a wide range of space applications like path-constrained proximity manoeuvers, large-angle attitude slew manoeuvers, autonomous rendezvous and docking, self assembly and on-orbit servicing, and spacecraft formation and station keeping. An APF based approach for autonomous spacecraft navigation and self-assembly in space can be found in [18], [19]. The operation of multi-spacecraft systems in three different potential fields is developed in [20], such that spacecraft formation can be held with as little effort as possible. A control scheme based on behavior-based control suitable for self-assembly and formation-flying applications is presented in [21]. A major inherent drawback of APF method is the existence of local minima that differ from the desired configuration. A method to avoid local minima using simulated annealing for local and global path planning of mobile robots is proposed in [22].

Spacecraft path planning would be successful only if the system is equipped with an efficient control system. Variable structure control with a sliding mode, first described in [23] and [24], laid out a well described robust control method. Today, Sliding Mode Control (SMC) is used as a general control method and is being examined for a wide spectrum of systems including nonlinear systems, multi-input/multi-output systems, discrete-time models, large scale and infinite-dimensional systems and stochastic systems. Numerous theoretical advances and practical applications have been reported in [25]-[29]. The methodology of using artificial potential field and sliding mode control for swarm aggregation and formation acquisition can be found in [30]. However, the SMC has not been applied widely for space applications. A technique for spacecraft path planning that exploits a behaviour-based approach to achieve an autonomous and distributed control of identical spacecraft over their relative geometry is presented in [31]. Also they prove that sliding mode control for spacecraft formation control is an effective way of implementing distributed architectures.

Reference [32] presents a novel hybrid propulsion using conventional electric thrusters and electrostatic forces generated by spacecraft charging in GEO. Satellite swarm aggregation based on hybrid propulsion is presented in [33]. This paper aims to further explore and enhance the applicability of SMC and APF algorithm presented in [30] and [33], for spacecraft formation flying missions. In this work, the actuation is performed by means of the hybrid propulsion system developed in [32]. The results presented in this paper prove that the use of APF and SMC with hybrid propulsion is an efficient method for path planning, control and actuation for spacecraft formations in GEO and other high Earth orbits. The outline of the paper is as follows: The fundamentals of the artificial potential field method and hybrid propulsion system are presented in Section II. The proposed path planning and control algorithms for formation flying are explained in Section III. The results of simulation study for a tetrahedron formation are included in Section V, followed by concluding remarks in Section V.

II. FUNDAMENTALS

A. Path Planning Using Artificial Potential Field

Let the formation consists of N individual agents in the n dimensional Euclidean space [30]. The position of the i^{th} agent is described by $\mathbf{x}_i \in \mathbf{R}^n$. It is assumed that synchronous motion exists and there is no time delay. The motion of each agent in the formation is governed by the equation:

$$\dot{\mathbf{x}}_i = \sum_{j=1, j\neq i}^{N} g(\mathbf{x}_i - \mathbf{x}_j), i = 1,\ldots,N, \tag{1}$$

where $g(.)$ is an odd function which represents the sum of the function of attraction and repulsion between the agents. The function $g(.)$ can be represented by $g(\mathbf{y}) = -\mathbf{y}[g_a(\|\mathbf{y}\|) - g_r(\|\mathbf{y}\|)]$ where $\mathbf{y} \in \mathbf{R}^n$ is arbitrary and $\|\mathbf{y}\| = \text{sqrt}(\mathbf{y}^T\mathbf{y})$ is the Euclidean norm. Equation (1) can be represented also by:

$$\dot{\mathbf{x}}_i = -\nabla_{x_i}\mathbf{J}(\mathbf{x}),\ i = 1,\ldots,N, \tag{2}$$

where $\mathbf{x}^T = \lfloor \mathbf{x}_1^T \quad \ldots \quad \mathbf{x}_N^T \rfloor$ is the lumped vector of the positions of all the agents and $\mathbf{J}:\mathbf{R}^{nN} \to \mathbf{R}$ is a potential function that represents the inter-individual interactions. The potential function $\mathbf{J}(\mathbf{x})$ depends on the relative positions of the agents in the formation. Under certain conditions the above model results in aggregation of the agents. In particular, it is needed that the attraction term $g_a(\|\mathbf{y}\|)$ dominates on large distances (needed for aggregation) and the repulsion term $g_r(\|\mathbf{y}\|)$ dominates on short distances (needed to avoid collisions) and there is a distance δ at which the attraction and the repulsion balance and $g_a(\delta) = g_r(\delta)$. Since there are no stochastics in the above model it is inferred that given the initial positions of the agents $\mathbf{x}_i(0),\ i = 1,\ldots,N$, the final configuration to which the agents will converge is unique. However, in general it is difficult to find a direct relation between $\mathbf{x}(0)$ and the final position $\mathbf{x}(\infty)$. This is a shortcoming of the above model in which all the agents interact with all the other agents in the same manner and there are no pair-dependent relationships. Therefore, the above model is more for general aggregation purposes instead of formation control.

For formation control, one needs to achieve and maintain a predefined geometrical shape (a formation) from possibly arbitrary initial positions of the agents. For this reason, the above equation needs to be modified, in order to be used for formation control. Using the assumptions on the potential function stated in [30], the equation of motion in (1) with the pair dependent attraction/repulsion becomes

$$\dot{\mathbf{x}}_i = \sum_{j=1, j\neq i}^{N} g_{ij}(\mathbf{x}_i - \mathbf{x}_j), i = 1,\ldots,N \tag{3}$$

where the attraction/repulsion function $g_{ij}(.)$ for all pairs (i, j) are odd functions and satisfy $g_{ij}(\mathbf{x}_i - \mathbf{x}_j) = -g_{ji}(\mathbf{x}_j - \mathbf{x}_i)$. For formation control, the attraction and repulsion functions, and therefore the equilibrium distances at which the attraction and the repulsion balance δ_{ij}, for different pairs of spacecraft can be different. In other words, for each pair (i, j) there is a distance δ_{ij} such that $g^a_{ij}(\delta_{ij}) = g^r_{ij}(\delta_{ij})$ (where $g^a_{ij}(\cdot)$ represents the pair-dependent attraction and $g^r_{ij}(\cdot)$ represents the pair-dependent repulsion) and depending on the formation requirements, it is possible to have $\delta_{ij} \neq \delta_{ik}$ for $k \neq j$. The desired formation can be uniquely specified with respect to rotation and translation by the formation constraints $\| \mathbf{x}_i - \mathbf{x}_j \| = d_{ij}$ for all $(i, j), j \neq i.$ The idea is to choose each of the attraction/repulsion functions $g_{ij}(.)$ such that $\delta_{ij} = d_{ij}$ for every pair of spacecraft (i, j). Then the corresponding potential function (or basically the generalized Lyapunov function)

$$\mathbf{J}(\mathbf{x}) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} [\mathbf{J}^{ij}_a(\| \mathbf{x}^i - \mathbf{x}^j \|) - \mathbf{J}^{ij}_r(\| \mathbf{x}^i - \mathbf{x}^j \|)],$$

has its minimum at the desired formation and once the formation is achieved $\dot{\mathbf{x}}_i = 0$ for all i.

One issue to note here is that this type of potential suffers from local minima problem mentioned before. However, we would like to stress that the procedure based on the sliding mode control method discussed in the following sections is not limited to this type of potentials only. In particular, if $\mathbf{J}(\mathbf{x})$ is chosen such that it has a unique minimum at the desired formation, then the desired formation will be asymptotically achieved for any initial condition. In the case of potentials with multiple local minima it is still guaranteed that the desired formation will be achieved, however this guarantee holds only locally.

B. *Coulomb Spacecraft Charging*

The objective of this section is to introduce the strategy for an efficient hybrid propulsion system developed in [32]. A navigation strategy cannot be implemented with a purely Coulomb-based control concept, as sufficient thrust cannot be produced when separation between individual satellites is large. General charge control strategies to control the relative motion of N satellites are still an active area of research. In particular, the required inter-satellite forces or equivalent product of satellite charges can be determined. However, how to effectively map these charge products into individual satellite charges is an open challenge. At this stage this analysis is still idealized and will be refined for particular charge implementation strategies in the future.

In this work, it is assumed that for a swarm of N satellites in GEO, having charge products $p = (N(N-1)/2)$, the charge products can be perfectly implemented into individual real satellite charges. For the i^{th} satellite, consider all possible pairs of charge products due to the remaining $N-1$ satellites as $Q_{ij} = q_i q_j$, $j = 1,\ldots,N, i \neq j$. Then the commanded force acting on the i^{th} satellites is:

$$\mathbf{u}^c_i = \sum_{j=1, i\neq j}^{N} (k_c Q_{ij} / \mathbf{x}_{ij}{}^2) \exp(-\mathbf{x}_{ij} / \lambda_d) \tag{4}$$

where $k_c = (4\pi \varepsilon_0)^{-1} = 8.99 \times 10^9$ Nm^2/C^2 is a constant of proportionality that depends on the permittivity of free space, $\mathbf{x}_{ij} = \| \mathbf{x}_i - \mathbf{x}_j \|$ is the satellite separation and λ_d is the Debye length. The commanded force is calculated using an appropriate control law. The aim is to design each of the control inputs such that the desired formation is achieved. To ensure this, it is necessary to enforce the condition in (2). In short, the control inputs are designed such that the velocity of the satellite is enforced along the negative gradient of the potential function $\mathbf{J}(\mathbf{x})$.

III. PROPULSION, PATH PLANNING AND CONTROL FOR SATELLITE FORMATION FLYING

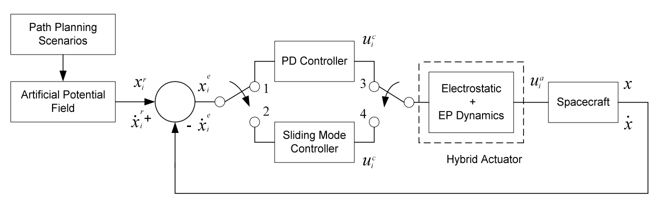


Fig. 1 Overall architecture of the proposed scheme

The schematic diagram of the proposed path planning, control and propulsion scheme is shown in Fig. 1. It integrates various sensory signals to achieve collision-free goal oriented navigation and formation control. The APF module is capable of avoiding obstacles and provides a goal-oriented navigation in an optimal time period. In the APF method, the dynamic environment in which the spacecraft exists is represented by a scalar potential function, which has a minimum potential (sink) at the desired terminal state of the vehicle dynamics and has maximum potential (source) at path constraints (near-by spacecraft or obstacles). In other words, the potential of the spacecraft formation will be constructed by identifying each spacecraft in the formation as a region of high potential. A repulsive force between them, which is simply the negative gradient of the potential field, avoids collisions between the neighboring spacecraft. Consequently, the spacecraft experiences a generalized force equal to the negative of the total potential gradient that drives the spacecraft towards the goal or the desired terminal state. Examples of source and sink fields are shown in Fig. 2. Since the rate of descent of the potential function is rendered negative definite, the potential-field approach guarantees that the spacecraft will converge to the desired terminal state without violating the defined path constraints. The convergence time depends on dissipative terms (damping) in the control law. The final formation will be achieved only if every member of the formation is in a sink corresponding to the final formation configuration.

This approach has less computational load as compared to other techniques that carry out extensive map building from raw sensory data at the expense of having the problem of local minima allowing only local results. The proposed navigation and control strategy will have all the benefits (and the drawbacks) associated with the APF method along with the added advantage of utilizing Coulomb forces.

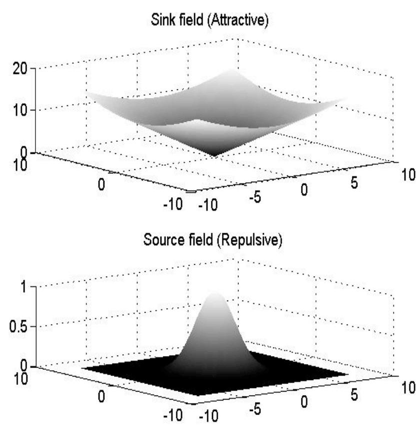


Fig. 2. Example of APF source and sink fields

The proposed PD and SMC systems would integrate with hybrid actuation using electrostatic forces and electric thrusters for high Earth orbits satellites. As shown in Fig. 1, the performances of the closed loop system can be evaluated by toggling between switches (1, 3) and switches (2, 4) for PD control and SMC respectively.

A. Sliding Mode Controller Design

Consider the general non-linear inertial equation of motion of the swarm agent represented by:

$$\mathbf{M}_i(\mathbf{x}_i)\ddot{\mathbf{x}}_i + f_i(\mathbf{x}_i,\dot{\mathbf{x}}_i) = \mathbf{u}_i^a, 1 \le i \le N \qquad (5)$$

where, $\mathbf{x}_i \in \mathbf{R}^n$ is the position vector of agent i, $\mathbf{M}_i \in \mathbf{R}^{n \times n}$ is the mass or inertia matrix and is assumed to be non-singular, $f_i(\mathbf{x}_i,\dot{\mathbf{x}}_i) \in \mathbf{R}^n$, N is the number of agents in the swarm, $\mathbf{u}_i^a \in \mathbf{R}^n$ is the control input. The additive term $f_i(\mathbf{x}_i,\dot{\mathbf{x}}_i)$ is assumed to be of the form

$$f_i(\mathbf{x}_i,\dot{\mathbf{x}}_i) = f_i^k(\mathbf{x}_i,\dot{\mathbf{x}}_i) + f_i^u(\mathbf{x}_i,\dot{\mathbf{x}}_i)$$

where $f_i^k(\mathbf{x}_i,\dot{\mathbf{x}}_i)$ represents the known part and $f_i^u(\mathbf{x}_i,\dot{\mathbf{x}}_i)$ is the unknown part of the system dynamics. Here $\mathbf{u}_i^a$ denote the actual force available from the hybrid thrusters to i th satellite for changing its maneuver. In the APF method, corresponding to (5), the motion of the individual agent is governed by (3). For the sliding mode control method, the n-dimensional sliding manifold for i^{th} swarm agent (here satellite) is chosen as:

$$\mathbf{s}_i = \dot{\mathbf{x}}_i + \nabla_{x_i}\mathbf{J}(\mathbf{x}) = 0, i = 1,\ldots,N \qquad (6)$$

Note that here the potential function $\mathbf{J}(\mathbf{x})$ is not static. It depends on the relative positions of the individuals in the swarm and need to satisfy certain assumptions made in [30]. Once all the satellite reach the respective sliding manifolds $\mathbf{s}_i = 0$, equation (6) reduces to $\dot{\mathbf{x}}_i = -\nabla_{x_i}\mathbf{J}(\mathbf{x})$ which is same as the motion (3) of the satellite swarm. A sufficient condition for sliding mode to occur given in [24] is satisfaction of:

$$\mathbf{s}_i^T\dot{\mathbf{s}}_i < 0, \forall i = 1,\ldots N \,. \qquad (7)$$

This guarantees that starting from any initial point in the state space; the sliding manifold is reached asymptotically. Further, if the condition

$$\mathbf{s}_i^T\dot{\mathbf{s}}_i < -\varepsilon \,\|\, \mathbf{s}_i \,\|, \forall i = 1,\ldots N$$

is satisfied, then it is guaranteed that sliding mode will occur in finite time. In order to achieve this objective, the sliding mode controller is given by:

$$\mathbf{u}_i^c = -\mathbf{u}_i^o\, sign(\mathbf{s}_i) + f_i^k(\mathbf{x}_i,\dot{\mathbf{x}}_i) \qquad (8)$$

where, $sign(\mathbf{s}_i) = \left[sign(\mathbf{s}_1) \;\; \ldots \;\; sign(\mathbf{s}_N)\right]^T$. The gain of the control input is chosen as $\mathbf{u}_i^o > (1/\underline{\mathbf{M}}_i)(\overline{\mathbf{M}}_i f_i + \mathbf{J} + \varepsilon_i)$, for some $\varepsilon_i > 0$, and (with this choice) it is guaranteed that $\mathbf{s}_i^T\dot{\mathbf{s}}_i < -\varepsilon_i \,\|\, \mathbf{s}_i \,\|$. Here $\underline{\mathbf{M}}_i$ and $\overline{\mathbf{M}}_i$ are the known lower and upper bounds of the inertia matrix respectively.

In the above controller, only the known part $f_i^k(\mathbf{x}_i,\dot{\mathbf{x}}_i)$ of the disturbance is considered. For practical implementations, a major inherent drawback of sliding mode controllers is the chattering phenomenon. Finite high frequency oscillations are generated due to the presence of unmodeled fast dynamics of the sensors and actuators and due to non-ideal realization of the relay characteristics of the SMC. In order to reduce the chattering phenomenon, the $sign(\mathbf{s}_i)$ term in the controller equation (8) can be replaced by a smooth approximation using $tanh(\beta\mathbf{s}_i)$. Note that this smoothing function does not guarantee full chatter elimination. It only ensures that the resulting sliding motion will lie in a close vicinity of the sliding manifold. There are other more advanced chattering elimination techniques using higher order sliding mode control. However, these techniques are out of the scope of this paper. The sliding mode controller used in this study is:

$$\mathbf{u}_i^c = -\mathbf{u}_i^o(x)tanh(\beta\mathbf{s}_i) + f_i^k(\mathbf{x}_i, \dot{\mathbf{x}}_i). \qquad (9)$$

Note that this controller is designed following the procedure in [30] and as if there is no actuator in the system. In other words, it is designed as if the control variable $\mathbf{u}_i^c$ in (9) is the control input $\mathbf{u}_i^a$ in equation (5). However, here the dynamics of the hybrid propulsion actuator are present as well and since they are not considered during the design of the controller (basically they are unmodeled dynamics) they may have negative effect on the performance of the system.

B. PD Controller Design

The PD controller for the i^{th} satellite can be expressed in terms of the position and velocity errors as:

$$\mathbf{u}_i^c = f_i^k(\mathbf{x}_i, \dot{\mathbf{x}}_i) + \underbrace{\mathbf{M}_i^o \ddot{\mathbf{x}}_i^r}_{I} \underbrace{- K_{vi}(\dot{\mathbf{x}}_i^r - \dot{\mathbf{x}}_i)}_{II} \underbrace{- K_{pi}(\mathbf{x}_i^r - \mathbf{x}_i)}_{III}, \qquad (10)$$

$\forall i = 1,\ldots,N$, where

$\mathbf{x}_i^r$ is the desired position,

$\mathbf{x}_i$ is the actual position of i^{th} satellite respectively

$\mathbf{M}_i^o$ is the nominal (known) mass of the satellite.

The parameters K_{pi} and K_{vi} are the proportional and the derivative gains, respectively, which are to be chosen by the designer based on the selected values of settling time and damping ratio. Here it is assumed that $\mathbf{u}_i^c$ in (10) is the control input $\mathbf{u}_i^a$ in (5). Note that this controller requires the knowledge of the second derivative with respect to time of the desired (reference) trajectory $\ddot{\mathbf{x}}_i^r$ and recall that the sliding mode controller did not require this information. Note also that, as in the sliding mode controller, the known part of the system dynamics $f_i^k(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ is utilized here as well. If there are no known dynamics in the system this part of the controller can be set to zero. With this choice of the controller the dynamics of the closed loop system, i.e., the dynamics in (5), become

$$\mathbf{M}_i(\mathbf{x}_i)(\ddot{\mathbf{x}}_i^r - \ddot{\mathbf{x}}_i) =$$

$$(\mathbf{M}_i(\mathbf{x}_i) - \mathbf{M}_i^o)\ddot{\mathbf{x}}_i^r + f_i^u(\mathbf{x}_i, \dot{\mathbf{x}}_i) + K_{vi}(\dot{\mathbf{x}}_i^r - \dot{\mathbf{x}}_i) + K_{pi}(\mathbf{x}_i^r - \mathbf{x}_i),$$
$$\forall i = 1,\ldots,N \qquad (11)$$

where, $f_i^u(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ is the unknown part of the system dynamics and the term $(\mathbf{M}_i(\mathbf{x}_i) - \mathbf{M}_i^o)\ddot{\mathbf{x}}_i^r$ is due to the uncertainty in the mass of the satellite. In other words, it is due to the fact that the actual mass of the satellite can be different from its known nominal mass. If the exact mass of the satellite is known then this part becomes zero. If these uncertainties ($f_i^u(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ and $(\mathbf{M}_i(\mathbf{x}_i) - \mathbf{M}_i^o)\ddot{\mathbf{x}}_i^r$) are different from zero, then they may have adverse effects on the performance of the system. The derivative gain K_{vi} is:

$K_{vi} = 2\mathbf{M}_i^o \zeta_i \omega_{ni}$, where $\omega_{ni} = \dfrac{4}{\zeta_i t_{si}}$,

ζ_i is the damping ratio and

t_{si} is the settling time for the i^{th} satellite. Note that for a fair comparison of the performance of PD controller with SMC, the same potential function defined by (2) is used here also. This implies that for the PD controller, the gradient of the potential in (2) represents the desired velocity $\dot{\mathbf{x}}_i^r$, its integral the desired position $\mathbf{x}_i^r$, and its time derivative $\ddot{\mathbf{x}}_i^r$. As in the design of the sliding mode controller, here the control variable $\mathbf{u}_i^c$ in (10) is designed as if it is the control input $\mathbf{u}_i^a$ in (5) without considering the actuator dynamics.

C. Formation Flying using Hybrid Propulsion

In this work, first the charge product is determined from the commanded force $\mathbf{u}_i^c$ and is then used to determine the actual electrostatic force that acts on each satellite in the swarm. At present, the control forces in (9) or (10) make no consideration for what forces can be implemented with Coulomb thrusting and which are not. Then using least-square inverse, the charge product $\tilde{\mathbf{Q}}$ for the i^{th} satellite can be computed from (4). The charge product thus derived is then used for computing the actual thrust developed by Coulomb charging of i^{th} satellite as:

$$\mathbf{u}_i^{CSF} = [\mathbf{A}_i][\tilde{\mathbf{Q}}] \qquad (12)$$

where $[\mathbf{A}_i] = [(k_c / x_{i1}{}^2) x_{i1} \exp(x_{i1}/\lambda_d)\ldots$
$(k_c / x_{iN}{}^2) x_{iN} \exp(x_{iN}/\lambda_d)]$

$[\tilde{\mathbf{Q}}] = [q_i q_j \quad \ldots \quad q_i q_N]^T$.

Note that this actual Coulomb force will generally not be equal to the commanded force. These formation internal forces cannot change the cluster momentum. Such force components are produced by the Electric Propulsion (EP) system by computing:

$$\mathbf{u}_i^{EP} = \mathbf{u}_i^c - \mathbf{u}_i^{CSF}. \qquad (13)$$

In other words, the actual thrust acting on i^{th} satellite is:

$$\mathbf{u}_i^a = \mathbf{u}_i^{CSF} + \mathbf{u}_i^{EP}. \qquad (14)$$

From (14), it is seen that the electric thrusters are used only for compensating the difference between the commanded force $\mathbf{u}_i^c$ and that generated by electrostatic forces $\mathbf{u}_i^{CSF}$. For scenarios where the Coulomb force is saturated due to large separation distances, or because the inertial cluster momentum must be changed, the EP thrusting will smoothly compensate and guarantee that the required navigation control force is always produced. A detailed derivation can be found in [32].

IV. Simulation Study

The goal of this section is to simulate a tetrahedron formation using APF method, Coulomb forces, PD and sliding mode controllers. The Snecma PPS 1350 EP thruster is used for simulation study. This electric thruster is suitable for operating over 5000 hours and has stable operation over a power range of 1200W to 1600W. Moreover, the starting power requirement is also low and it suits for the applications considered in this work. Consider the dynamic model in (5). The effect of gravity and other orbit dynamics are neglected for simulation purpose. The satellites are assumed to be floating freely in deep space. It is assumed that a slowly time varying (24 hour period) differential solar radiation perturbation with a 2 micro-Newton magnitude is included. This is known type of perturbation in high Earth orbits. The magnitude is assumed here based on the analysis presented is [35]. Such small forces often have a negligible effect on a large formation. However, for very close cluster formation flying even this small force can cause drifts of hundreds of meters of a 24 hours time period. A detailed discussion on differential perturbation can be found in [35]. It is assumed that both the uncertainty in the satellite mass and external perturbation are same for both SMC and PD controller design. The bounds of the uncertainty in mass are set as ±50% of the nominal mass of the satellite. The bound on the known disturbance is set to $2\mu N$.

Consider the scenario where four satellites form a tetrahedron formation. For formation control, each satellite in the formation is pre-assigned to a desired position in the final formation. The potential function considered for formation control problem is a function with linear attraction and exponential repulsion terms, and is given by:

$$g(y) = -y(a - b\exp(-\| y^2 \| / c), \qquad (15)$$

where a, b and c are positive constants such that $b > a$. The constant a is the magnitude of the attraction and b is the magnitude of the repulsion and the constant c is its spread or repulsion range, but the actual repulsion is some combination of the effects of both.

The parameter a is computed in order to achieve the balance of attraction and repulsion between any two satellites at the desired distance d in the final tetrahedron formation. Let $a = b\exp(-d^2/c)$, $b = 5\times10^{-5}$, $c = 100$ and $d = 10$. For formation control, each agent in the formation is pre-assigned a desired relative position in the final formation. By increasing the repulsive force (i.e., by increasing b), it is possible to avoid collisions. By equating (15) to zero, it can be seen that $g(y)$ switches sign at the set of points defined by:

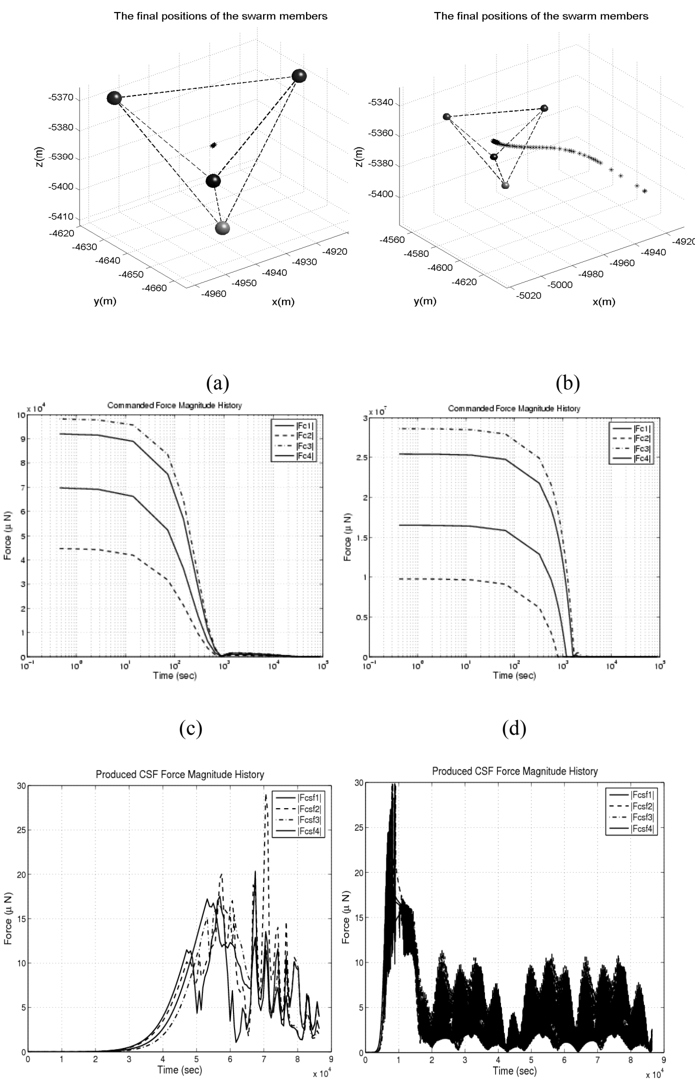
$$\Psi = \{g(y) = 0 \text{ or } \| y \| = \delta = sqrt(c\ln(b/a))\}.$$

The distance at which attraction balances the repulsion is given by $\delta = sqrt(c\ln(b/a)) = 10$ which is equal to the desired distance d. For simulation purpose using PD control, the settling time is chosen as 70 sec and the damping ratio is chosen as 1. The proportional gain K_{pi} is unity and the derivative gain K_{vi} used for simulation is 17.1.

TABLE I
Simulation Parameters

Satellite parameter	Value/Units
Individual mass	150kg
Bounds on mass	±50%
Satellite diameter	0.5m
Debye length (GEO)	200m
Charge saturation limit	2μC
Number of satellite	4
Manoeuvre time	24hrs
Peak magnitude of differential disturbance (GEO)	2μN
Peak thrust of Snecma PPS 1350 electric thruster	88,000 μN
Specific impulse of Snecma PPS 1350 electric thruster	1650 sec
Power (nominal) of Snecma PPS 1350 electric thruster	1500 W
Mass of Snecma PPS 1350 electric thruster (including 2 Xe flow control systems)	5.3 kg

The simulation plots for SMC and PD control are shown in Fig. 3. It is assumed that initially the satellites are at rest with an average inter-satellite separation of around 2.7 km. With time, the four satellites move to their required final inter-satellite separation of 10m and form the required tetrahedron formation while avoiding collisions. Note that for this particular formation there is no local minima and therefore the local minima problem inherent in the potential functions method is not present here (and the formation can be achieved globally). The objective in this study is to compare the properties of the PD and SM controllers and not to test the effectiveness of the potential functions method. Fig. 3 (a) and (b) show the final formation positions and the center of the formation is represented by '*' for SMC and PD control respectively. From these plots, it is observed that the formation center movement is more with PD control compared to that with SMC method. One of the reasons for the center movement is presence of external perturbations. However, with SMC the performance is far better due to the inherent invariance or robustness properties of the method. The center movement can lead to delay in achieving the final formation. The control inputs to the four satellites generated by SMC and PD controller are shown in Fig. 3 (c) and (d) respectively. It is observed that the control effort is much more with PD than with SM. The thrust generated by Coulomb charging is showed in Fig. 3 (e) and (f).



(a) (b)

(c) (d)

(e) (f)

Fig. 3. Simulation results for tetrahedron formation: SMC controller (a), (c) and (e); PD controller (b), (d) and (f)

V. Conclusion

This paper presents the application of an algorithm for autonomous path planning and control of spacecraft formations with hybrid propulsion using artificial potential field method and sliding mode control. The simulation results show that for the scenarios considered in this study, the control effort required by the individual satellite is far less with sliding mode controller when compared to that using PD controller. It is observed that the swarm center movement is less with sliding mode control facilitating quicker achievement of the formation and hence greater fuel saving. Moreover, sliding mode controller is the inherent insensitivity to parameter variations and disturbances once in the sliding mode, thereby eliminating the necessity of exact spacecraft modeling. Simple chatter elimination techniques are used to make this practical for a tetrahedron spacecraft formation-flying scenario. The use of hybrid propulsion helps to save fuel and hence reduce the mission cost and thereby shows the advantage of using this novel algorithm for GEO and other high Earth orbit spacecraft formations. It is believed that these preliminary results give an insight to the problem of controlling satellite formations. The feasibility of this approach for various other formation scenarios has been successfully validated in simulation and has been generalized for N number of spacecraft to demonstrate micro-spacecraft swarm aggregation scenarios. However, more work is needed to fully exploit electrostatic forces to make the concept of electrostatic propulsion a reality for a real deep space mission.

In recent years, there has been increasing interest in close-proximity (10-100m) spacecraft formation flying missions in the Low Earth Orbit (LEO). Contrary to GEO, every individual charged spacecraft in LEO experiences the Lorentz force due to its interaction with the Earth's magnetic field and is not a result of interaction amongst charged spacecraft. The direction and magnitude of the Lorentz force is determined by the spacecraft position and orbital geometry. The application of this new concept of using Lorentz force for spacecraft formation in LEO is presented in [36].

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