Feasibility of Using Lunar Magnetic Fields to Control CubeSat Attitude

by

James Andrew Penrod

Other Degrees: N/A

An undergraduate thesis submitted to the

Faculty of the Graduate School of the

University of Colorado in partial fulfillment

of the requirements for the degree of

Bachelor of Science

Department of Aerospace Engineering Sciences

2015

This thesis entitled:
Feasibility of Using Lunar Magnetic Fields to Control CubeSat Attitude
written by James Andrew Penrod
has been approved for the Department of Aerospace Engineering Sciences

---

Professor Hanspeter Schaub

Jeffrey Parker

Scot Douglass

Date ____________

The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

Penrod, James Andrew (B.S., Aerospace Engineering Sciences)

Feasibility of Using Lunar Magnetic Fields to Control CubeSat Attitude

Thesis directed by Professor Hanspeter Schaub

Magnetorquers represent a promising method for controlling CubeSat attitude. Both magnetorquers and CubeSats are used primarily in Earth orbit. Magnetorquers can be useful for controlling spacecraft attitude due to their low mass, volume, and power requirements, and therefore represent an intriguing option for future CubeSat missions beyond Earth. A first-order analysis is presented on the feasibility of controlling CubeSat attitude in lunar orbit using a magnetorquer to de-spin the spacecraft about one axis. This first-order analysis examines the time required to de-spin the spacecraft from 1°/s to 0.01°/s for a variety of altitudes, inclinations, and eccentricities, using LADEEs orbital trajectory as a baseline. Due to the weak and erratic nature of the moons magnetic fields, the de-spinning process is possible, but requires a sufficiently long time of roughly 50 days depending on the exact orbit.

Dedication

To Jake and Zack in honor of your courage and perseverance in the face of adversity. May you seek truth and know joy.

Acknowledgements

Thanks to Dr. Schaub for his guidance and technical knowledge in both researching this topic and writing this paper.

Contents

Chapter

Figures

Figure

Chapter 1

Introduction

1.1 Background and Motivation

CubeSats are small satellites, usually around 1 L in volume, designed to accomplish a specific task in orbit[1]. They have become popular in the last decade because they are inexpensive to manufacture and launch. So far, CubeSats have primarily been considered for applications in Low Earth Orbit (LEO), though Vermont Technical College has entered NASA's CubeQuest program to launch a CubeSat into lunar orbit.[1]

As a fully functional spacecraft, each CubeSat contains every subsystem of a larger satellite, but their small size presents a major technical challenge for spacecraft designers. Traditional methods of attitude control, like reaction wheels and gas thrusters, occupy valuable volume that could otherwise be used for scientific payloads or telecom packages. Many CubeSats instead use a magnetorquer for attitude control, because magnetorquers take up much less space than other attitude control methods[2].

Magnetorquers have an extensive history of attitude control in LEO, but have never been used in lunar orbit. The moon's magnetic fields are weaker and more irregular than Earth's dipole magnetic field, but a magnetorquer could in theory control attitude even in the erratic lunar environment[3]. This project investigates the feasibility of de-spinning a lunar-orbiting CubeSat using magnetorquers. It was inspired by an effort to replicate the Lunar Atmosphere and Dust Environment Explorer (LADEE) mission using a constellation of CubeSats. [2]

[1] Vermont Technical College CubeSat Laboratory, http://www.cubesatlab.org/ [Accessed 2015-03-28]

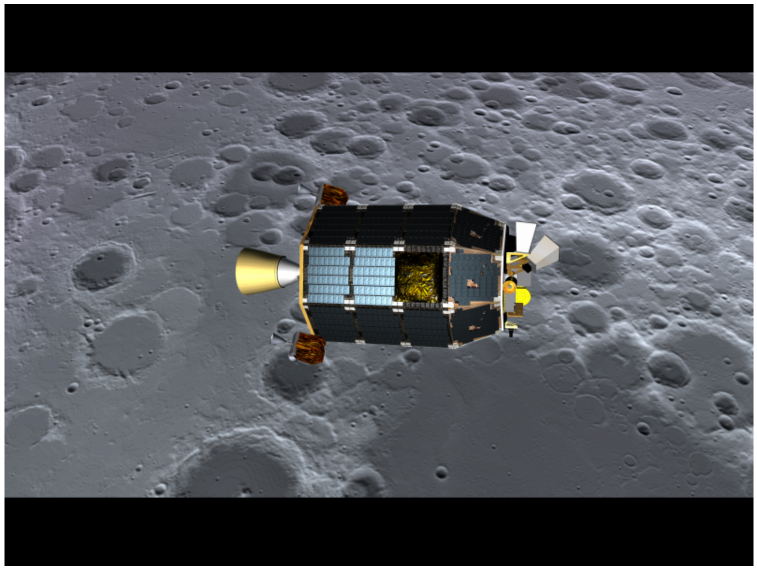


Figure 1.1: NASA LADEE spacecraft in lunar orbit

This paper provides a first-order approximation on the length of time required for the CubeSat to reach a stable attitude. This analysis first models lunar magnetic fields using spherical harmonics. Then the spacecraft's position in orbit relative to the moon is calculated and the magnetic field at that location is determined. The analysis collects magnetic field data along the spacecraft's path and integrates over time to determine the spacecraft's new attitude. For this first order approximation, the model will determine the time required for a satellite to de-spin from 1 °/s to 0.01 °/s using only the moon's magnetic fields, neglecting an influence from terrestrial or solar magnetic fields. The primary emphasis will compare different altitudes, inclinations, and eccentricities to a baseline orbit modeled on LADEE's orbit. The model assumes that the spacecraft is oriented such that the torque applied by the magnetic field at each point in the orbit is maximized. As such, this analysis produces the minimum time necessary for CubeSat de-spin to prove feasability. The conclusion discusses methods for future work whereby this model can be refined.

[2] Image Credit: NASA Ames / Dana Berry, http://www.nasa.gov/content/ladee-above-the-lunar-surface-artists-concept/

1.2 History of Magnetorquers in Orbit

Earth's magnetic fields have affected spacecraft attitude since Vanguard I in 1958, which was de-spun by 2.5 rps in its first two years in orbit. In 1960, Transit 1B intentionally used the Earth's magnetic fields and an on-board dipole to de-spin while aligning its axis of symmetry along the Earth's magnetic field lines[2]. Full 3-axis attitude control using magnetorquers was theoretically demonstrated by Wang, Shtessel, and Wang in 1998[4]. The development of CubeSats in the new millenium has increased demand for a low-mass, low-power, low-maintenance attitude control system, and magnetorquers have frequently been enlisted to meet that need. CubeSats using magnetorquers for attitude control include the University of Colorado's DANDE[3] and Stanford University's Gravity Probe B[4] . Companies like Andrews Space[5] and Zarm Technik[6] have developed lines of magnetorquers for a range of spacecraft applications and Zarm in particular specializes in magnetometers and magnetorquers. So far, magnetorquers have primarily been considered for Earth-orbiting spacecraft.

1.3 Research Overview and Scope

This project arose out of a goal to replicate the LADEE mission using a constellation of lunar CubeSats and represents an initial analysis for CubeSat attitude control in lunar orbit. As a first order analysis, several simplifying assumptions were made to establish feasibility. The goal of the research is to estimate the time required to de-spin a CubeSat in lunar orbit using magnetorquers; that is, full 3-axis attitude control is not considered here. Specifically, the objective of this research is to quantify the amount of time required to de-spin a CubeSat from 1°/s to 0.01°/s. The analysis is conducted assuming the spacecraft is a 10cm x 10cm x 10cm cube spinning only around its major axis, so the same spacecraft face points towards nadir at all times.

---

[3] Colorado Space Grant Consortium, http://spacegrant.colorado.edu/about-dande/spacecraft [Accessed 2015-03-28]

[4] Stanford University Gravity Probe B, https://einstein.stanford.edu/TECH/technology2.html [Accessed 2015-03-28]

[5] http://andrews-space.com/torque-rods/ [Accessed 2015-02-04]

[6] http://www.zarm-technik.de/ [Accessed 2015-02-04]

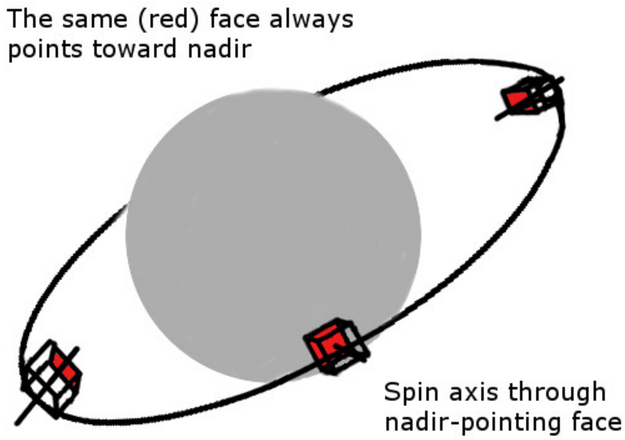


Figure 1.2: Orientation of the CubeSat, where the same face always points toward nadir

The spacecraft is assumed to be homogeneous, so the center of gravity is located at the center of the cube, and the magnetorquer passes through the spacecraft center of gravity. The moon is considered a point mass for its gravitational fields so that two-body conditions apply. The magnetorquer is assumed to work ideally and interact perfectly with the lunar magnetic fields without interference from any on-board components. Finally, the moon's magnetic field is assumed to be constant with time, and no effects from Earth's magnetic field or the sun are considered.

The goal of this research is to estimate the time required to de-spin the CubeSat. The scope of the project is to analyze differing values for semi-major axis, eccentricity, and inclination, while keeping magnetorquer strength constant. The project uses the LADEE mission as a baseline and uses Zarm Technik's 10 cm magnetorquer for reference[7] .

[7] http://www.zarm-technik.de/downloadfiles/ZARMTechnikAG_CubeSatTorquers_web2010.pdf [Accessed 2015-02-04]

1.3.1 Modelling Lunar Magnetic Fields

Earth's moon is geologically inactive, so its core no longer generates a magnetic dynamo. Remnant magnetism exists in patches of the moon's crust, but they do not follow the familiar dipole pattern of Earth's magnetic field[5]. Instead, the lunar magnetic field is weak and erratic. While Earth's magnetic field strength is roughly 50,000 nT, the moon's magnetic field strength sporadically reaches only 100 nT[6]. The moon's magnetic fields, unlike the Earth's, cannot be modeled as a planetary dipole, but spherical harmonics present a way to model the magnetic field vectors throughout the CubeSat's orbit. Spherical Harmonics represent the Legendre Polynomials that are a solution to Laplace's Equation. The coefficients for the Legendre Polynomials must be collected experimentally. This report uses correlated spherical harmonics coefficients that Purucker and Nicholas of NASA Goddard Space Flight Center compiled using data from the Lunar Prospector mission[3].

1.3.2 Orbit Propagation

An orbit can be fully defined using Kepler's five orbital elements: semi-major axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), and argument of perigee (ω). Right ascension and argument of perigee locate the plane of the orbit in inertial space. For this analysis, only the spacecraft position relative to the lunar surface is considered, so right ascension and argument of perigee are set as constant. More precisely, right ascension of the ascending node is replaced by longitude of the ascending node, and is set to $0°$, and argument of perigee is also set to $0°$. This project varies a, e, and i, while keeping Ω and ω fixed above the near side of the moon. After the orbit is defined, the spacecraft location within the orbit is determined by its true anomaly, ν, which varies non-linearly with time. The orbit is propagated through time using Kepler's Equation to determine ν as time increments. With the spacecraft position defined in time, the Keplerian Element Set can be converted to give the spacecraft's location within a Cartesian or Spherical coordinate system with an origin at the center of the moon. This allows for the magnetic field vector to be calculated at each spacecraft position.

1.3.3 Magnetorquer Sizing

The Magnetorquer is how the spacecraft interacts with the moon's magnetic fields. Magnetorquers are electromagnets that use a set of perpendicular coils to control 3-axis attitude. When a current is passed through the coil, the magnetorquer produces a magnetic field that interacts with the external field to create a moment on the spacecraft. The size of the moment created depends on the strength of the external field, the number of coils, and the current through the coils[7]. For this analysis, it is assumed that the magnetorquer works ideally and no consideration was given to the battery size required to power the magnetorquer. However, the magnetorquer itself was subject to size constraints: each coil can be no longer than 10 cm so that it fits in a 1U CubeSat.

Chapter 2

Modelling Lunar Magnetic Fields

2.1 Research Goal

The moon's magnetic field can be approximated as a combination of spherical harmonics, which are the three dimensional solutions to Laplace's Equation. The goal of this project is to create in MATLAB a working 3-dimensional model of the moon's magnetic field using sperheical harmonics. The model will be validated when it produces a 3D representation of the moon's magnetic fields that match Mark Wieczorek's image of the Lunar Prospector data.

2.2 Magnetic Field Models

The two main magnetic field models are the dipole model and the spherical harmonics model. The dipole model imagines the body as a single large magnet with a north and south pole. Magnetic field lines run parallel to each other from the south pole to the north pole and the field is strongest near the surface of the body and near the poles. Earth and the Sun have active internal dynamos and can be roughly modelled as dipoles. The spherical harmonic model poses a spherical isosurface for the magnetic field with irregular protrusions where the magnetic field is stronger. The size and shape of the protrusions are governed by a set of coefficients and resemble the electron shells within an atom. Since the moon has irregular patches of magnetism dotting its surface, its magnetic field is best modelled using spherical harmonics.

2.3 Spherical Harmonics Model

Spherical harmonics were introduced by Laplace in 1782 and are solutions to Laplace's Equation[8]:

$$\nabla^2 \phi = 0 \tag{2.1}$$

The magnetic field vector can be calculated from the gradient of the local potential:

$$\mathbf{B} = -\nabla V \tag{2.2}$$

As stated by Davis[9], V is given by

$$V(r,\theta,\phi) = a \sum_{n=1}^{k} (\frac{a}{r})^{n+2} \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\theta) \tag{2.3}$$

Here, r, θ, and ϕ are spherical coordinates, a is the radius of the moon, g and h are experimentally derived coefficients, and $P_n^m(\theta)$ is the Schmidt quasi-normalized associated Legendre Polynomial. Legendre Polynomials are orthogonal functions that satisfy Laplace's Equation and can be calculated using Rodrigues' formula

$$(1 - 2vx + x^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(v) x^n \tag{2.4}$$

Associated Legendre Polynomials are related to normal Legendre Polynomials by

$$P_{n,m}(v) = (1 - v^2)^{1/2m} \frac{d^m}{dv^m}(P_n(v)) \tag{2.5}$$

Normalization is the process that uses a set of linearly independent functions to make an orthogonal basis with respect to a weighting function. There are two common normalization methods: Gaussian normalization and Schmidt quasi-normalization. Historically, Schmidt quasi-normalization has been used for magnetic field modelling[3], a convention that will be followed here. The normalized Legendre polynomial is related to the associated Legendre polynomial by

$$P_n^m = [\frac{2(n-m)!}{(n+m)!}]^{1/2} P_{n,m} \tag{2.6}$$

The magnetic field vector is calculated using these normalized associated Legendre Polynomials:

$$B_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^{k} (\frac{a}{r})^{n+2}(n+1) \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\theta) \tag{2.7a}$$

$$B_\theta = \frac{-1}{r}\frac{\partial V}{\partial \theta} = -\sum_{n=1}^{k} (\frac{a}{r})^{n+2}(n+1) \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) \frac{\partial P_n^m(\theta)}{\partial \theta} \tag{2.7b}$$

$$B_\phi = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial \phi} = \frac{-1}{\sin\theta} \sum_{n=1}^{k} (\frac{a}{r})^{n+2}(n+1) \sum_{m=0}^{n} (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\theta) \tag{2.7c}$$

Note that g_n^m and h_n^m are also normalized using Schmidt quasi-normalization[9].

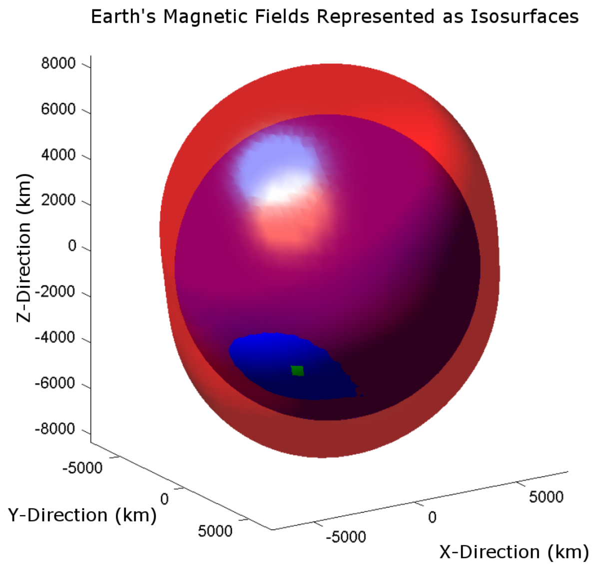
2.3.1 NASA Lunar Prospector Dataset

Lunar Prospector was launched in 1998 and orbited the moon for over a year at very low altitudes. During its extended mission, Lunar Prospector was lowered to an altitude of 30 km, which allowed it to get much higher resolution of the moon's magnetic and gravitational fields.[1] Purucker and Nicholas from the Goddard Space Flight Center present the full set of Schmidt-normalized coefficients, which were used in this project.

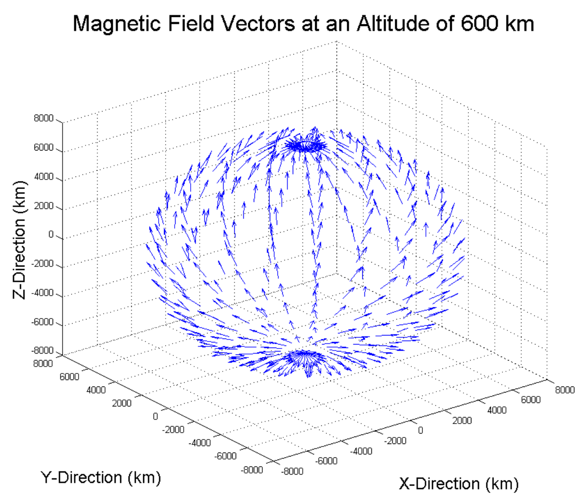
2.3.2 Implementation in MATLAB

The algorithm for implementing Purucker and Nicholas' coefficients to produce a model of the moon's magnetic fields borrows heavily from the algorithm and code developed by Jeremy Davis. Both are designed to load Schmidt-normalized spherical harmonic coefficients. Thus, the code simply increments through n and m to sum the values of B_r, B_θ, and B_ϕ for the values of r, θ, and ϕ specified by the user in the main function.The code converts the magnetic field from spherical coordinates to Cartesian coordinates and plots the resulting magnetic field vectors on a 3-D plot. The implementation is verified because the resulting image matches the image produced by Davis for Earth's magnetic field. Davis performed his own validation and the model is further validated for the lunar data by inspection with the image produced by Wieczorek.

[1] LADEE mission and trajectory design, http://www.spaceflight101.com/ladee-mission-and-trajectory-design.htm [Accessed 2015-03-03]



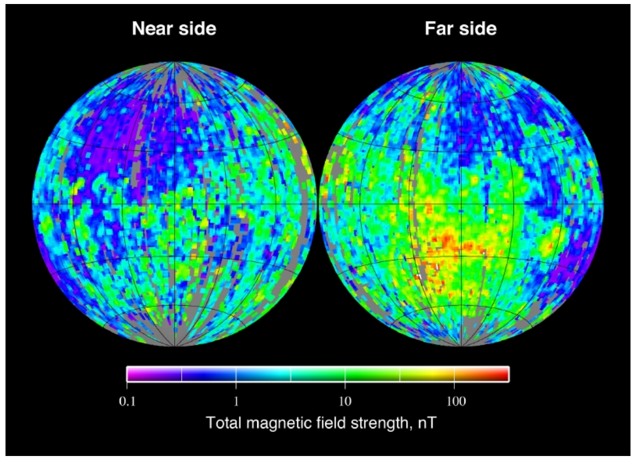
(a) Jeremy Davis' Isosurface Plot of Earth's Magnetic Field[9]



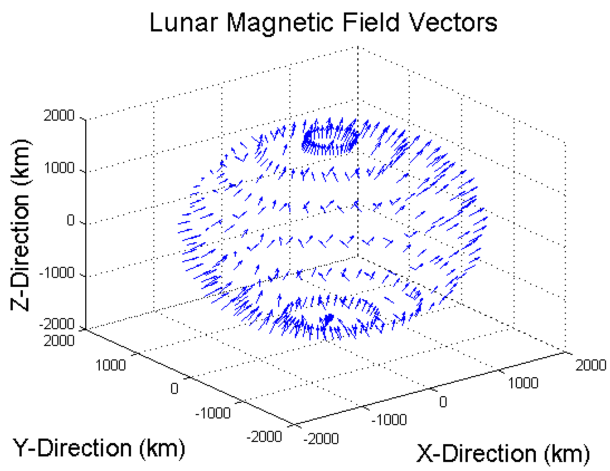
(b) Code-generated vector field of Earth's Magnetic Field

Figure 2.1: Comparison between Davis magnetic field and code-generated magnetic field at 600 m

As expected, the magnetic field vectors around the Earth run primarily parallel to the Earth's surface and are stronger at the poles. Closely inspecting the magnetic field magnitude shows that the minimum magnitude occurs just south of the equator, where the South Atlantic Anomaly occurs. The general code architecture is validated, but the specific case using Purucker's coefficients needs to be verified as well.



(a) Mark Wieczorek's plot lunar surface magnetic field strength



(b) Code-generated vector field of Lunar Magnetic Field at an altitude of 0 km

Figure 2.2: Comparison between Wieczorek lunar magnetic field and code-generated lunar magnetic field at 0 km

The moon's magnetic fields are expectedly erratic and generally point away from the lunar surface. On the right side of the image, the vectors are noticeable longer than anywhere else, corresponding to the strong magnetic field on the far side of the moon seen on Wieczorek's plot.[2] The magnitude of the magnetic field vectors ranges from approximately 2 nT to 24.5 nT. This is slightly less than what is shown in Wieczorek's plot, but the difference is because the code-

[2] http://upload.wikimedia.org/wikipedia/commons/3/37/Moon_ER_magnetic_field.jpg Used with permission. [Accessed 2015-03-01]

generated plot has fewer data points and less resolution. It is likely that it did not sample the highly magnetic peaks visible in Wieczorek's plot. Nevertheless, the general structure of the field and the higher magnitude vectors clustered on one side of the sphere verify that the MATLAB code runs properly.

Chapter 3

Lunar Orbit Parameters and Spacecraft Location

3.1 Research Goal and Assumptions

Spherical harmonics determine the strength of the moon's magnetic field at all points in lunar orbit. Analyzing the magnetic torque acting on the spacecraft at a given time requires knowing where the spacecraft is relative to the center of the moon. This analysis uses the same selenocentric-fixed orbital frame used to orient the spherical harmonics.

3.2 Orbit Propagation Using Kepler's Equation

In this first-order analysis, the moon and CubeSat are assumed to be in a two-body relationship. This assumes that the mass of the spacecraft is negligable, the coordinate system is inertial, the spacecraft and the moon are spherically symmetric and uniformly dense, and no other forces act on the spacecraft or the moon[10]. The moon's gravity field is treated as a point mass and the J_2 effect is ignored. These assumptions are made to focus on the attitude of the satellite, rather than the magnetic field effects on the orbit itself. The Keplerian orbital elements that define the orbit are: semi-major axis, eccentricity, inclination, longitude of the ascending node, argument of perigee, and true anomaly. Longitude of the ascending node and argument of perigee are set to 0°for simplicity. The analysis uses LADEE's orbit[1] as a baseline:

$a = 1791$ km

[1] LADEE-Mission and Trajectory Design, http://www.spaceflight101.com/ladee-mission-and-trajectory-design.html [Accessed 2015-03-03]

$e = 0.0091$

$i = 157°$

Subsequently, e is incremented by 0.055 from 0 to 0.5, i is incremented by 10°from 0°to 90°, and a is incremented from 10 km to 10,000 km in 10 logarithmic increments. For each orbit, the spacecraft is given an inital rotation rate of 1°/s and is de-spun until it reaches 0.01°/s. At the beginning of the simulation, ν is set to 0. Time is incremented by 60 seconds and ν is calculated again using Kepler's Problem. Kepler's Problem uses Mean Anomaly to determine the spacecraft position in time. True anomaly is related to mean anomaly via eccentric anomaly. Eccentric anomaly is calculated by:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2} \tag{3.1}$$

Eccentric Anomaly is related to Mean Anomaly by:

$$M = E - e\sin E \tag{3.2}$$

As time increments, the new Mean Anomaly is calculated by

$$M = M_0 + \sqrt{\frac{\mu}{a^3}}\Delta t \tag{3.3}$$

Note that μ is the gravitational parameter of the moon, 4902.799 km^3/s^2[10]. The new True Anomaly is then determined by calculating the Eccentric Anomaly that corresponds to the new Mean Anomaly and the True Anomaly that corresponds to the new Eccentric Anomaly. The Eccentric Anomaly must be calculated numerically since Equation 3.2 is non-linear, but when E is known then ν can be calculated by solving Equation 3.1 for ν. All six Keplerian elements are now defined for each increment of t and so the spacecraft's inertial position in cartesian coordinates can be calculated, which is necessary to correlate the position of the spacecraft to the local magnetic field. First, the position vector is calculated in the perifocal frame[10]:

$$\boldsymbol{r}_{PQW} = \begin{bmatrix} \frac{p\cos\nu}{1+e\cos\nu} \\ \frac{p\sin\nu}{1+e\cos\nu} \\ 0 \end{bmatrix} \tag{3.4}$$

Here, p is the semi-parameter, which is defined as

$$p = a(1 - e^2) \tag{3.5}$$

The perifocal frame is transformed into the inertial coordinate system with a transformation matrix as follows:

$$\boldsymbol{r}_{XYZ} = \begin{vmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\sin\omega\cos i & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i \\ \sin\omega\sin i & \cos\omega\sin i & \cos i \end{vmatrix} \boldsymbol{r}_{PQW} \tag{3.6}$$

3.3 Implementation in MATLAB

A main program was written to cycle through the values for a, e, and i. The main program calls a subfunciton that calculates the spacecraft position with ν starting at 0. For each time iteration, ν is updated using Kepler's Problem as described above, using Newton-Raphson iteration to solve for Eccentric Anomaly from Mean Anomaly numerically. True Anomaly is updated, and the spacecraft position is updated and the process begins again with a new time increment. Intially, this continued for a user-specified time duration, but it was eventually updated to stop whenever the spacecraft spin rate was below 0.01°/s, as discussed in Chapter 5. The final version of the code, designed to save time, propagates around 1 orbit and extrapolates the time to de-spin based on the deceleration experienced over the one orbit. The main function went through inclination first, then eccentricity, then semi-major axis. It produced three sets of 3-D graphs, representing a vs e, a vs i, and e vs i.

Chapter 4

Magnetorquer Sizing

4.1 Research Goal and Assumptions

Finally, this investigation analyzes how the lunar magnetic fields interact with the CubeSat as it orbits the moon. In this spacecraft architecture, the primary interface between the CubeSat and the lunar magnetic fields occurs in the magnetic torque bars.

4.2 Magnetorquer Operation and Types

There are two main methods for controlling spacecraft attitude using magnetorquers. The spacecraft attitude can be controlled directly through the interaction between the magnetorquer and the environment's magnetic fields, or the magnetorquers can be used to desaturate on-board reaction wheels. Each method has its advantages. The reaction wheel method can use smaller magnetorquers and therefore less power to maintain its attitude and is able to absorb larger attitude perturbations more easily. Direct control has less mass and mechanical complexity because it does not utilize reaction wheels. It also actively controls spacecraft attitude instead of correcting perturbations from a desired attitude and is therefore the method that is used for this analysis.

The most common architecture for direct attitude control using magnetorquers is to mount a set of three torque rods orthogonally so that each one aligns with one of the spacecraft axes. Each of the rods is wired to create a magnetic field in either direction, depending on the direction of the current, so that a magnetic field can be generated in any direction. This will be utilized for this analysis, by assuming that the magnetorquer magnetic field is oriented orthogonal to the lunar magnetic field at all times. The torque acting on the spacecraft is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B} \tag{4.1}$$

Here, τ is the torque on the spacecraft, μ is the magnetorquer magnetic field, and B is the local lunar magnetic field. The magnetic field produced by the magnetorquer is found from Ampere's Law to be:

$$B = \frac{\mu N I}{L} \tag{4.2}$$

B is the magnetic field, oriented along the direction vector of the magnetorquer, μ is the permeability of the magnetorquer material, N is the total number of coils in the magnetorquer, I is the current through the wire, and L is the total length of the magnetorquer. Zarm Technik Magnetorquers are an industry standard for magnetorquers and have been used on many CubeSats and more traditional spacecraft, including GOCE and GRACE, so all values are taken from Zarm's datasheets on Magnetic Torquers for Micro-Satellites. Zarm magnetorquers use a Nickel-alloy core with a permeability of $4.4 \times 10^{-4} H/m$. Their 10 cm magnetorquer model takes a current of 55 mA. Zarm's 10 cm magnetorquer has a magnetic dipole moment of $0.5Am^2$ [1] . While varying orbital parameters, this value is kept constant.

4.3 Implementation in MATLAB

The theoretical CubeSat has one magnetorquer corresponding to each axis, allowing for the magnetic field to be oriented in any direction. The spacecraft is assumed to be rotating around its major axis but otherwise fixed in inertial space, so the magnetorquer field is configured such that the resulting torque vector is along the major axis of the spacecraft as well. The dipole moment is assumed to be constant at $0.5Am^2$ in this analysis, though it would vary between $0.5Am^2$ and $0.707Am^2$ as the spacecraft rotated. This is implemented in the parameters of the main file while the orbital parameters are varied. Since only 1-axis attitude control is considered for this feasibility

[1] Magnetic Torquers for Micro-Satellites, http://www.zarm-technik.de/downloadfiles/ZARMTechnikAG_CubeSatTorquers_web2010.pdf [Accessed 2015-02-04]

study, this analysis provides an upper bound on the time required to de-spin. A full 3-axis attitude control system using magnetorquers would require significantly more time to de-tumble.

Chapter 5

Full Model Implementation and Results

All of the analysis in this investigation is done in MATLAB. While varying orbit parameters, the magnetorquer was as strong as possible, taking up the full 10 cm allocated to it. An industry-standard magnetorquer can have 250,000 turns in its coil and carry a current of 63 mA[7], leading to a magnetic moment of $0.5Am^2$. This value was kept constant while varying the orbital parameters and was used to calculate the moment acting on the satellite.

The main function in the code iterates through values of a, e, and i. For each combination, it sets the initial value of ν to 0 and calculated the initial inertial position of the spacecraft. It calculates the magnetic field at that location using the subfunction derived from code developed by Jeremy Davis. This allows the torque on the satellite to be calculated. Since the CubeSat is assumed to be a 10 cm x 10 cm x 10 cm homogeneous cube, the moment of inertia is calculated by

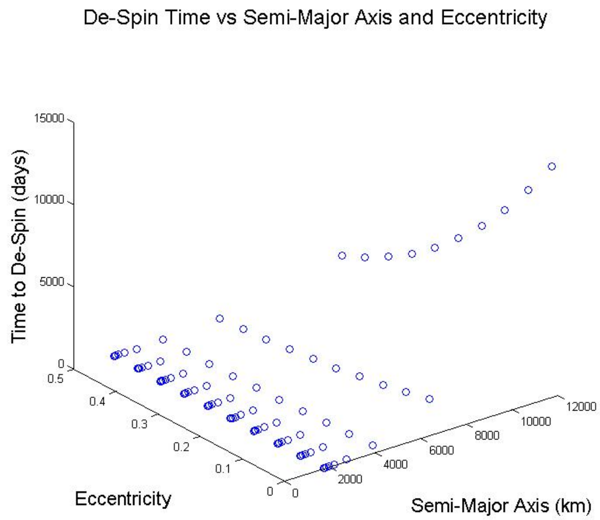
$$I = \frac{ml^2}{6} \tag{5.1}$$

The mass of the CubeSat is assumed to be 1.33 kg, the maximum allowed for a CubeSat[1]. The resulting moment of inertia is calculated to be $0.0022kgm^2$ and the angular deceleration of the spacecraft is calculated using this value by

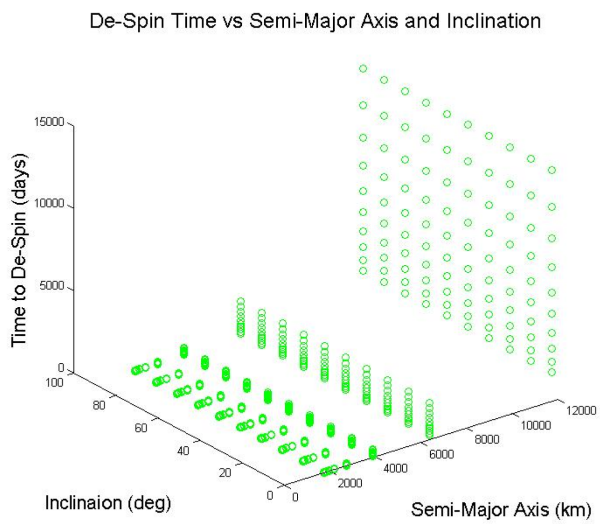
$$\tau = I\alpha \tag{5.2}$$

Time is incremented by one minute, over which the angular acceleration is assumed to be constant. The spacecraft's angular velocity is updated and the true anomaly is updated using Kepler's Problem. The process then begins again with the inertial position being calculated. This process is repeated over the course of one orbit and the final angular velocity is compared to the initial angular velocity. The time required to de-spin is calculated by multiplying the time to complete one orbit by the number of orbits required to decrease angular velocity from 1°/s to 0.01°/s. It is assumed that there are no perturbing forces on the spacecraft so the moment it experiences from the moon's magnetic fields is constant between orbits. Once the time to de-spin is calculated, inclination is incremented by 10°. Once the ten inclination values have been calculated, eccentricity is inremented by .056. Finally, semi-major axis is incremented logarithmically from 1747.4 km (10 km altitude) to 11737.4 km (10,000 km altitude). The output is a matrix that contains the time to de-spin in the first column and the corresponding orbital elements in the 2-4 columns. Then, MATLAB's 3-D plotting function, plot3, is used to plot the time to despin vs a and e, a and i, and e and i.

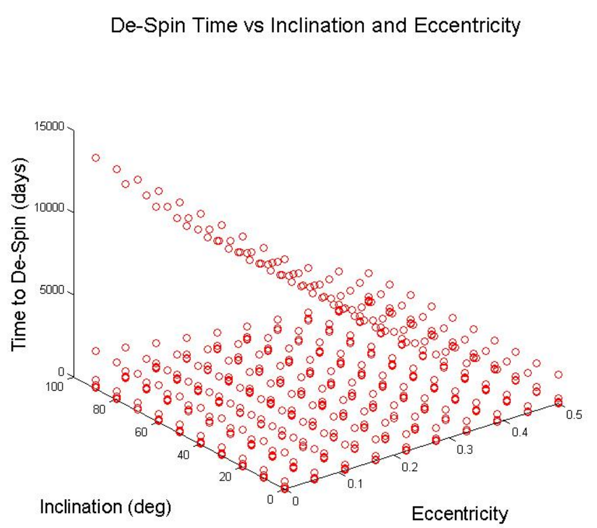
These plots show that the time to de-spin is heavily influenced by the spacecraft's distance from the lunar surface. Small values of a and large values of e result in the shortest time to de-spin. The shortest time to de-spin is $5.02 \times 10^5 s$ (about 5.8 days), which occurs when $a = 10$ km and $e = 0.5$. Unfortunately, the periselene for this orbit is lower than the surface of the moon. The optimal orbit analyzed that stays above the lunar surface is when $a = 1837.4$ km and $e = 0.11$ and the time to de-spin is $4.0107 \times 10^6 s$ (about 46.4 days). The minimum time to despin a spacecraft in a feasible orbit is in the same order of magnitude up to $a = 3891.8$ km, where $t = 64.2$ days, but when a jumps to 6379.0 km, $t = 282.6$ days. Using the LADEE orbit baseline, the spacecraft will despin in 48.6 days, which is reasonably close to the minimum de-spin time.



(a) Time to Despin vs a and e



(b) Time to Despin vs a and i



(c) Time to Despin vs e and i

Figure 5.1: Variation of time to despin vs orbital parameters

Chapter 6

Discussion and Conclusion

6.1 First-Order Feasibility

The data suggest that de-spinning a spacecraft using magnetorquers and lunar magnetic fields is feasible. Using magnetorquers that already exist in a homogeneous CubeSat, it takes around 50 days to de-spin from 1°/s to 0.01°/s, depending on the exact orbit. The response is too slow to provide full 3-axis attitude control at all times, but could be utilized in initial de-spin procedures, provided sufficient time is allowed. It should also be noted that this first-order analysis represents the ideal de-spin case and in practice the time required will likely be even greater than 50 days. Moreover, since other methods of attitude control must be used to maintain full 3-axis control, those same methods would likely be preferable for the de-spin stage of the mission as well, even though de-spinning using magnetorquers in lunar orbit is feasible. However, should the LADEE mission be recreated using CubeSats with magnetorquers, the LADEE orbit can be used with nearly optimal results. In the first-order feasibility, the LADEE orbit only takes 5 hours longer to de-spin than the optimal orbit. Future work should be done to analyze the full 3-axis attitude control for both the LADEE orbit and the optimal orbit.

6.2 Comparison to Other Forms of Attitude Control

Currently, reaction wheels are the most common method of attitude control in satellites around the moon. LADEE, which this project was meant to emulate, used reaction wheels to achieve 3-axis stabilization and the SFL CanX CubeSat in Earth oribt also used reaction wheels.

SFL's reaction wheels can achieve a torque of 2 mNm[11]. Assuming this moment from the reaction wheel is constant over time and as the satellite slows down, these 2 mNm reaction wheels can de-spin a 10x10x10 homogeneous CubeSat from 1°/s to .01°/s in 0.99s, six orders of magnitude quicker than the fastest possible magnetorquer de-spin. Small on-board chemical thrusters can achieve similar results, but these are rarely used on CubeSats because of the volume required for the propellant tanks.

6.3 Future Work

Three orthogonal magnetorquers at the center of gravity of a CubeSat do not provide an advantage over existing technologies for CubeSats in lunar orbit. However, a potential lunar CubeSat mission in the future may be volume constrained and find that reaction wheels occupy too much volume. Additional work could be done to determine whether the external casing of the CubeSat can be used as a magnetorquer system. This would restore the space advantages magnetorquers typically provide. However, more work still should be done before launching a magnetorquer-controlled CubeSat to lunar orbit. In particular, this project assumed the initial attitude of the CubeSat was oriented to align with the moon's magnetic fields. Future work should analyze the impact of different initial spacecraft attitudes, using a Monte Carlo simulation to determine which initial attitudes are likely to be encountered. This analysis discussed de-spinning the satellite in 1-D to provide a first-order feasibility study. Future work should investigate full 3-axis detumble feasibility. Work should also be done to analyze the spacecraft orbit more precisely. This analysis did not consider the moon's J_2 effect or other gravity perturbations, and work should also be done to investigate the presence of magnetic perturbations on the spacecraft's orbit.

Bibliography

[1] Lee, S., Hutputanasin, A., Armen, Toorian, A., Lan, W., and Munakata, R., "CubeSat Design Specification," CubeSat Design Specification, 2009.

[2] "Spacecraft Magnetic Torques," NASA Guidance and Control Design Criteria Document SP-8018, March 1969.

[3] Purucker, M. E. and Nicholas, J. B., "Global spherical harmonic models of the internal magnetic field of the Moon based on sequential and coestimation approaches," Journal of Geophysical Research, Vol. 115, 2010, pp. 401–409.

[4] Ping Wang, Y. S. and Wang, Y. Q., "Satellite attitude control using only magnetorquers," Thirtieth Southeastern Symposium on System Theory, Morgantown, West Virginia, 1998.

[5] Reuell, P., "Magnetic Anomalies on Moon," http://scitechdaily.com/magnetic-anomalies-on-moon/, Accessed: 2014-11-19.

[6] Mitchell, D., Halekas, J., Lin, R., Frey, S., Hood, L., Acuna, M., and Binder, A., "Global mapping of lunar crustal magnetic fields by Lunar Prospector," Icarus, Vol. 194, 2008, pp. 401–409.

[7] Celena Dopart, Robart Morlath, E. O. and Schomaker, J., "Design and Analysis for a CubeSat Mission," Major Qualifying Project Report, 2012.

[8] MacRobert, T. M., Spherical harmonics: An elementary treatise on harmonic functions, with applications, Pergamon Press, 1967.

[9] Davis, J., "Mathematical Modeling of Earth's Magnetic Field," Technical Note, May 12 2004.

[10] Vallado, D. A., Fundamentals of Astrodynamics and Applications, Science Technology Library, 2007.

[11] Doug Sinclair, C. C. G. and Zee, R. E., "Enabling Reaction Wheel Technology for High Performance Nanosatellite Attitude Control," Proceedings of 21st Annual AIAA/USU Conference on Small Satellites, 2007.

[12] Harland, D. M. and Lorenz, R., Space Systems Failures, Springer Science and Business Media, 2007.

[13] Richmond, N. C. and Hood, L. L., "A Preliminary Global Map of the Vector Lunar Crustal Magnetic Field Based on Lunar Prospector Magnetometer Data," Journal of Geophysical Research-Planets, Vol. 113, 2008.

[14] "Vermont Technical College CubeSat Laboratory," http://www.cubesatlab.org/, Accessed: 2015-03-28.

[15] "Magnetic Torquers for Micro-Satellites," Online datasheet http://www.zarm-technik.de/downloadfiles/ZARMTechnikAG_CubeSatTorquers_web2010.pdf, Accessed: 2015-02-04.