AN EXPLORATION INTO OPTIMAL MULTI-SPHERE METHOD SPHERE POPULATION FOR ELECTROSTATIC INTERACTION

Himanshi Singhal[*], Trevor Bennett[†], and Hanspeter Schaub[‡]

The Multi-Sphere Method provides an ability to model electrostatic interaction between spacecraft in orbit. However, no automated method exists for the sphere distribution which motivates exploration into sphere distribution algorithm research. This study distributes spheres over a representative solar panel to investigate several proposed methods of optimal sphere placement. The optimum is defined by the smallest error between the capacitance computed using the proposed methods and finite element method(FEM) values. Proposed are the various distribution methods, wave, simple grid and nested box method. Simple grid method involves a large number of spheres making it unreliable. Nested box method seems to be the most promising method as it has a sphere distribution more representative of the FEM charge distribution. It further allows more control over the setup parameters such as density of nested boxes, which makes it better than others. Some equations are still in development for this method. To pursue this method, this is broken down into a smaller version and is discussed as single box method, which will later be further modified to have multiple nested boxes.

INTRODUCTION

The growing defunct satellite and orbital debris population poses a threat to current and ongoing Earth-orbiting infrastructure. Therefore, it is of paramount interest to consider methods for reducing/re-orbiting the debris. Active debris removal (ADR) is a primary strategy for reducing the amount of orbital debris. Several methods such as harpoons, nets, and electrostatic interaction are considered.[2] This study further investigates into the electrostatic interaction removal strategy. The FEM yields better accuracy while taking more computational time. Therefore surface Multi Sphere Model(MSM) is used to enable faster than real-time implementation of electrostatic interactions keeping the accuracy and computation time in mind.[4] This approximates the electrostatic interactions between the rigid spacecraft bodies as a collection of spherical conductors dispersed throughout the respective craft.[4] A rigid spacecraft is shown with a collection of spheres dispersed in figure 1.

There are various challenges involved when it comes to using MSM. For instance, having more number of spheres adds more fidelity to the system simultaneously making it more computationally expensive. Therefore, placement of spheres is really an important aspect of study to get the best possible solution using the method. This study investigates several sphere distribution methods to get the most approximate sphere distribution to the real charge distribution. In general, spacecraft

[*]Undergraduate Research Assistant, Aerospace Engineering Sciences, University of Colorado
[†]Graduate Research Assistant, Aerospace Engineering Sciences, University of Colorado
[‡]Associate Professor, Department of Aerospace Engineering Sciences, University of Colorado,431UCB, Colorado Center for Astrodynamics Research, Boulder, CO 80309-0431

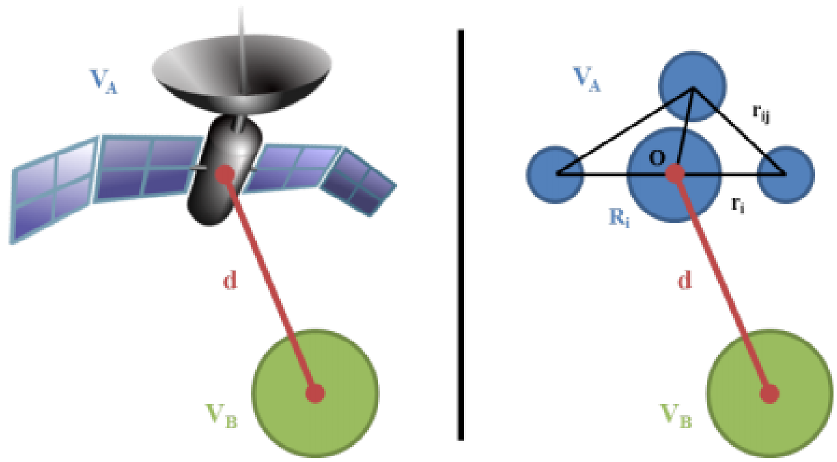

Figure 1. Conceptual Depiction of Multi Sphere Method[4]

have rectangular solar panels. The simple geometry and widespread use of a rectangular solar panel provides a suitable test geometry for the proposed sphere population optimization and is considered here. Prior work by Stevenson and Schaub show that sphere population on a sphere are achieved by the golden spiral.[4] Using the golden spiral, the optimal packing parameter γ is shown to converge to a value of 0.8454, which is used as the constant value in all the methods explored in this study. Constant packing parameter reduces the system complexity and makes it feasible to find the radius sphere R using equation 1. This study is a first exploration into generic sphere distribution characteristics and provides an initial foundation for sphere distribution automation.

ERROR AND OPTIMALITY

The performance measure for the proposed distribution methods is the difference between the computed capacitance compared to the FEM capacitance. The relative error determines how good a method is relative to FEM. In order to be as close as possible to the real charge distribution, the relative error is desired to be within the range of $\pm 5\%$. For comparison, the capacitance of a solar panel with a length of $l = 3$ meters, a breadth of $b = 1$ meters, and a thickness of $t = 0.01$ meters is given as $76.4678 * 10^{-12}$ Farad using the Maxwell 3D electrostatic FEM software package.

METHODS

The controls of the system depend on the electrostatic forces and torques acting on it, the individual charge q_i is required to compute using different sphere distributions. The charge on each sphere is found using the prescribed electrostatic voltage and individual sphere radius R. Equation 2 is used to find the charge q_i on each individual sphere where the sphere radius R is found using the packing parameter γ and n in equation 1. Note that V_A is the prescribed voltage on all the spheres in the model while the external sphere is held at V_B. The method does not have the capability of varying the voltages as it increases the complexity of the system. Also, keeping the voltage constant is consistent with modeled conducting sphere which will be kept at a uniform and constant voltage of 30 kV. The value of coulomb's constant k is $8.99 * 10^9$ $N.m^2/C^2$. The other parameter which is used in all the methods is the packing parameter and the value chosen for this is $\gamma = 0.8454$ for all the methods.[4] This is defined as the ratio of the area occupied by the total sphere distribution and the area of the object. The chosen value for γ is the converged optimal value for the Golden Spiral Method and is kept constant in the current setup of algorithm.

$$R = \sqrt{\frac{LB\gamma}{n * 4\pi}} \quad (1)$$

$$V_i = \frac{k_c q_i}{R_i} \quad (2)$$

The capacitance of the solar panel is computed using the charges q_i on all the spheres and the prescribed voltage. The equation[4] used to find capacitance is

$$V = C^{-1} q \quad (3)$$

where,

$$C^{-1} = \begin{pmatrix} \frac{1}{R_1} & \frac{1}{r_{1,2}} & \cdots & \frac{1}{r_{1,n}} & \frac{1}{r_{1,B}} \\ \frac{1}{r_{2,1}} & \frac{1}{R_2} & \cdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{r_{n,1}} & \cdots & \cdots & \frac{1}{R_n} & \frac{1}{r_{n,B}} \\ \frac{1}{r_{B,1}} & \cdots & \cdots & \frac{1}{r_{B_n}} & \frac{1}{R_B} \end{pmatrix} \quad (4)$$

In C^{-1}, the distance is defined as $r_{i,B} = d - r_i$.[4]

A variety of methods were developed to look more into distinct charge distribution methods to find the best possible computationally economic solution. All the methods are discussed in detail below.

Wave method

The wave method consists of distributing spheres on a sine/cosine wave within the object. Given an input number of waves and spheres, the individual sphere radius is obtained using Eq. 5 with a packing parameter of $\gamma = 0.8454$. The number of spheres n is defined by the user. Equations 6 and 7 are used to find the x and y coordinates of the system using the number of waves n_{wave} and panel dimensions. Figure 2 shows the sine curve with 10 waves.

$$R = \sqrt{\frac{LB\gamma}{n * 4\pi}} \quad (5)$$

$$x = 0 : \frac{L}{n} : L \quad (6)$$

$$y = \frac{1}{2} B \sin\left(\frac{2\pi x n_{wave}}{L}\right) \quad (7)$$

This method yields the capacitance relative error of about 4% with 40 spheres on 10 sine waves keeping the error within bounds. However, this method does not model the real charge distribution well as it is unable to capture the corner and edge effects of charging. This method had very few

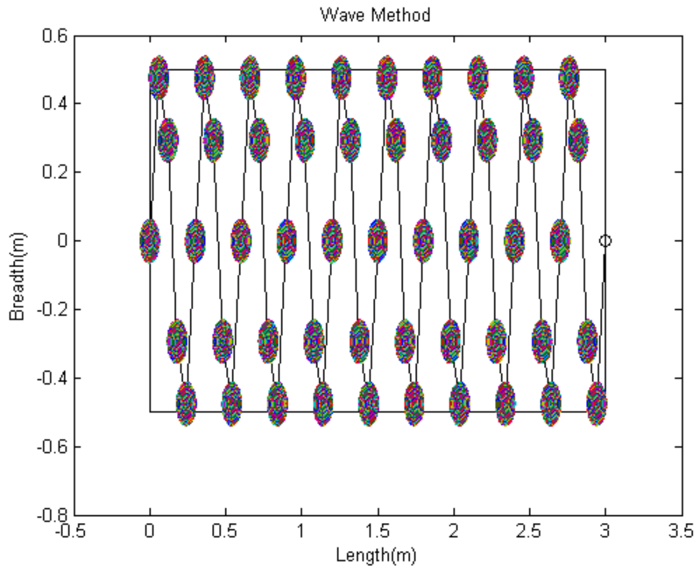

Figure 2. Distribution of spheres over the solar panel using the wave method

input parameters which reduced the complexity of the system substantially. Therefore it helped in gaining an insight into how different distribution can still give the values within the acceptable range. Figure 3 shows the error plotted with respect to the number of spheres onto the panel. The results from this method lead to different methods to explore the behavior of the distribution patterns.

Simple Grid method

Using the insight gained by the wave method, a new method was developed that utilizes a grid distribution. This method consists of making a grid on the panel. Given the initial number of rows n_r and columns n_c as the input parameters, number of spheres n is approximated as the product of $n_r * n_c$. Similar to the wave method, the radius then is defined using Eq. 8. Since it is a simple grid inside the panel, the equal spacing is defined as Eqs 9 and 10 .

$$R = \sqrt{\frac{LB\gamma}{n * 4\pi}} \tag{8}$$

$$\delta x = \frac{L}{n_c} \tag{9}$$

$$\delta y = \frac{B}{n_r} \tag{10}$$

The algorithm designed to do this computation is automated to change the number of rows, columns and spheres based on the feedback error it gets from the simulation. However, it is not fully automated to do everything on its own as of now and is one of the future goals of this research. Therefore, it allows the user to choose any number of rows, columns and spheres. The algorithm can update these numbers based on the relations mentioned above in Equations 8, 9, 10 to get the

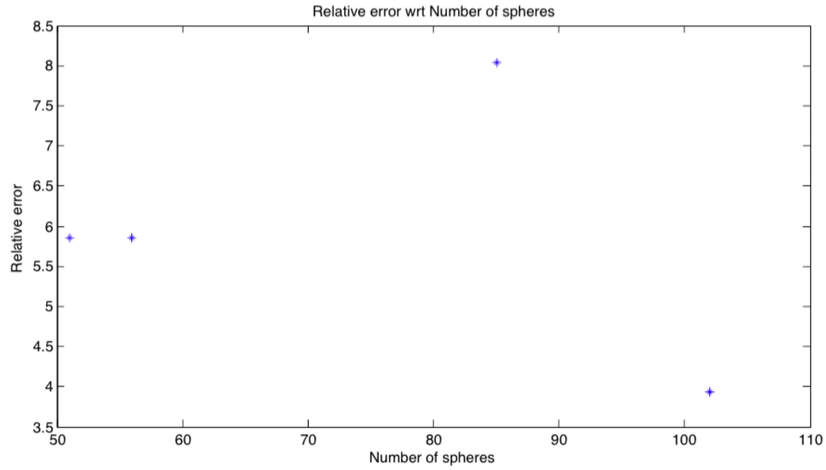

Figure 3. Relative error with respect to Number of spheres for wave method

right set of numbers to produce the relative error within the acceptable range. Figure 4 illustrates the fashion in which the grid is formed.

Figure 5 shows the relative error plotted against the number of spheres. It is very evident from the plot that it is possible to be within the maximum error range which is a plus for this model. But, the drawback this model is the large number of spheres. This method takes somewhere between 250 to 300 spheres to be very close to -5%, which is a lot of spheres on a 3m*1m panel. Having these many number of spheres on a small panel like that makes the method very inefficient and computationally expensive.

Nested Box method

Again, after getting unsatisfactory results from the simple grid method either, the next method developed was nested box method. As the name indicates, it consists of multiple boxes nested together in a panel. This method seems very promising as it can account for a lot of parameters to model the panel. But since it had a lot of parameters to deal with, it increased the complexity of the problem beyond our ability to handle at once. Therefore, this problem has been broken down into small chunks such as modeling a single box inside the panel for now. This is where the research is currently at. Once the single box method works, then it can be used to replicate to model multiple nested boxes with varying separation between them.

Single Box method This method uses only one box inside the panel for modeling it. Similar to above methods, this also consists of finding the radius of the spheres using the number of spheres n as an input to the system.

$$R = \sqrt{\frac{LB\gamma}{n * 4\pi}} \tag{11}$$

Since the algorithm is not fully automated, n is redefined using the sets of equation given below. Here k_b and k_l are the packing parameter for the breadth and length respectively, which both are chosen to be 0.84 as of now. It can be changed in the simulation if needed. The current work in

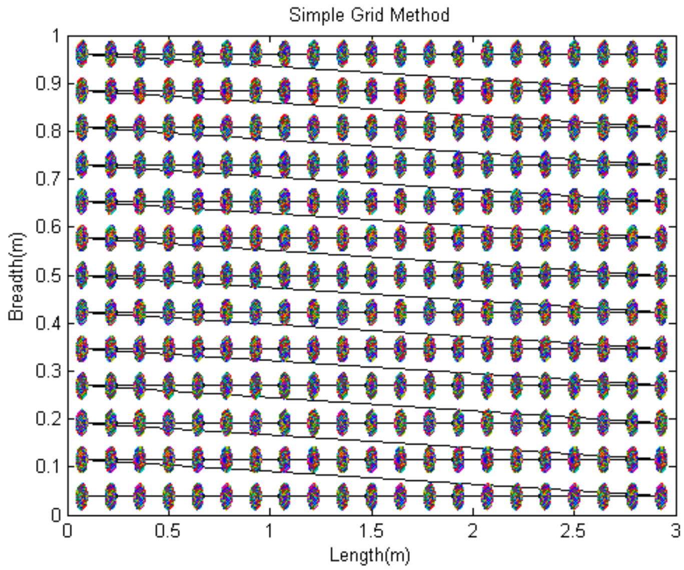

Figure 4. Distribution of spheres over the solar panel using the Simple Grid Method

this method includes finding the optimum k_b and k_l values to optimize the system results. Variables $n_{s,b}$ and $n_{s,l}$ defines the number of spheres on each length and breadth of the box. Hence sum of twice of each $n_{s,b}$ and $n_{s,l}$ gives the new total number of spheres it should have given the radius and dimensions of the box.

$$n_{s,b} = \frac{B * k_b}{2R} \tag{12}$$

$$R = \sqrt{\frac{L * k_l}{2R}} \tag{13}$$

$$\mathrm{n}_{new} = 2(n_{s,b} + n_{s,l}) \tag{14}$$

Therefore, the new number of spheres is updated in the algorithm using the difference between the initial number of spheres and the number of spheres found by the equations. This is repeated until the relative error is within the given range.

Figure 6 shows a box inside the solar panel with the updated number of spheres. Figure 7 is the plot of relative error against the number of spheres. This plot shows that changing the number of sphere merely by 1 affects the relative error by a significant amount and converging to a smaller value. These results seems promising because replicating this for multiple nested boxes with varying separation distances between them would help enable to model the real charge distribution on the panel, which is more concentrated on the outer sides and edges of the box.

Multiple Boxes Once the single box method works and outputs the results within error bounds, the method can be modified to accommodate multiple nested boxes. The method is planned to have the capability of controlling the distance between the nested boxes to replicate the real charge distribution. Since the charges accumulate on the corners and edges of an object, the distribution

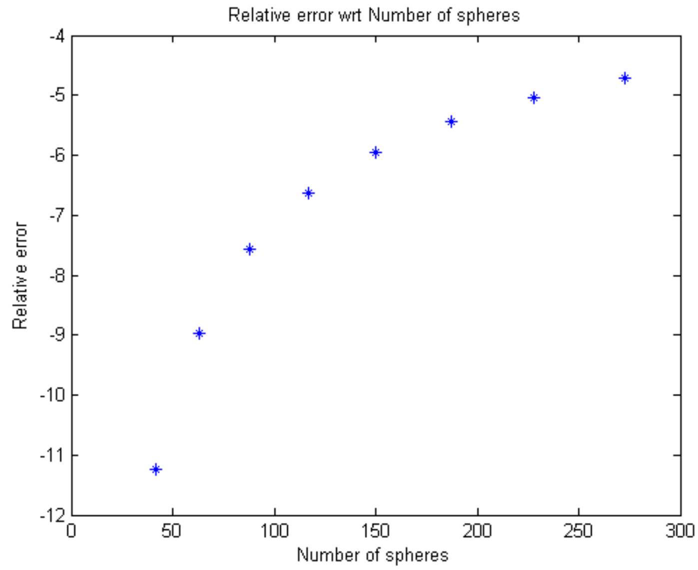

Figure 5. Relative error with respect to Number of spheres for simple grid method

will be denser at the edges and sparse in the center. Below are some proposed equations that will be used for this method in future. The input parameters to this system will be the number of spheres n and dimensions of the panel. Using γ, the sphere radius R can be found using Eq 1. The model will have an even number of boxes making it easier to deal with in pairs. The pairs are defined as outer box being the odd numbered box and inner box being the even numbered box in each pair. The number of spheres that will be placed on each box is found using the equations 15 and 16. The number of boxes n_{boxes} then are defined using the the total number of spheres using equation 17, where n is the sum of n_{outer} and n_{inner}. The method is still in development and therefore other plotting parameters aren't fully defined yet. But it will all be done in a similar fashion to the single box method.

$$\text{n}_{outer} = 2(L + B) \tag{15}$$

$$\text{n}_{inner} = 2(L + B - 1) \tag{16}$$

$$\text{n}_{boxes} = \frac{n}{4(L + B) - 2} \tag{17}$$

CONCLUSION

In an attempt to find a better charge distribution model to be as close as possible to the FEM value while keeping few parameters constant, three more methods were developed and were looked more into in details. One of the goals kept in mind throughout the process was to keep the system as simple as possible by having least number of spheres, input parameters etc. The wave method is the simplest way but it's simplicity costs on the other hand. It does not have the capability of modeling the real charge distribution. Although this method did not replicate real charge distribution but still

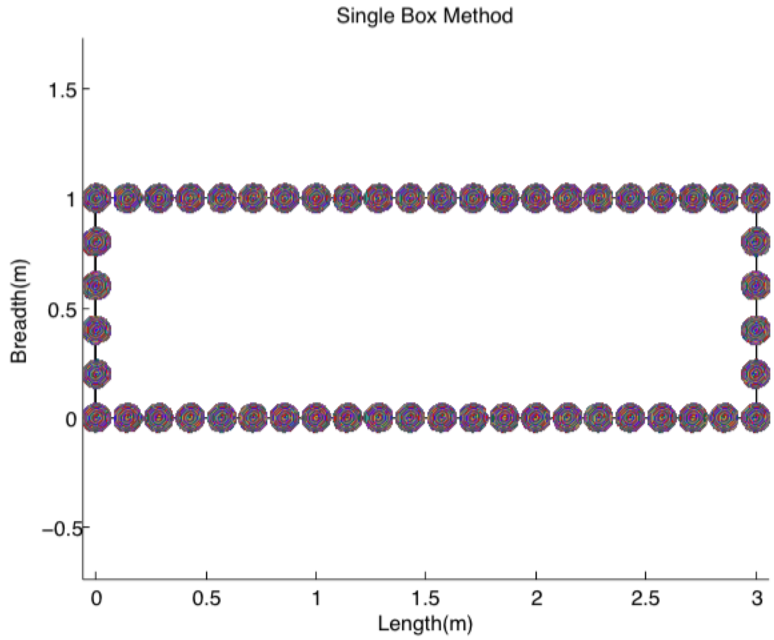

Figure 6. Distribution of spheres over the solar panel using the Single Box Method

gave a very useful insight into the distribution pattern keeping the relative error within the desired range. The simple grid method is the next simplest method which gave good results in terms of the error range but the drawback it had was the enormous number of spheres making this method inefficient and expensive. The third proposed method is the nested box method, which currently seems the most promising method as it accounts for a variety of parameter controls simultaneously increasing the complexity of the system. Therefore, this is broken down into a simple problem currently dealing with only one box which later can be replicated to have multiple nested boxes. The single box produced good results and the nested box method is expected to produce better results with all the parameters accounted in to optimize the system. Future work of this research is to develop the nested box method followed by making the algorithm more automated if this method gives the charge distribution model close enough to FEM within the error bounds. If not, then keep looking into more ways of charge distribution to come up with the best possible method.

REFERENCES

[1] L. E. Z. Jasper, C. R. Seubert, H. Schaub, V. Trushlyakov, and E. Yutkin, "Tethered Tug for Large Low Earth Orbit Debris Removal," *AAS/AIAA Spaceflight Mechanics Meeting*, Charleston, South Carolina, Jan. 29 – Feb. 2 2012. Paper AAS 12-252.

[2] E. Hogan and H. Schaub, "Space Debris Reorbiting Using Electrostatic Actuation," *AAS Guidance and Control Conference*, Breckenridge, CO, Feb. 3–8 2012. Paper AAS 12–016.

[3] T. Bennett and H. Schaub, "Touchless Electrostatic Three- Dimensional Detumbling of Large Geo Debris," *AAS/AIAA Spaceflight Mechanics Meeting*, January 26-30 2014.

[4] D. Stevenson, "Optimization of Sphere Population for Electrostatic Multi Sphere Model," *12th Spacecraft Charging Technology Conference*, Kitakyushu, Japan, May 14–18 2012.

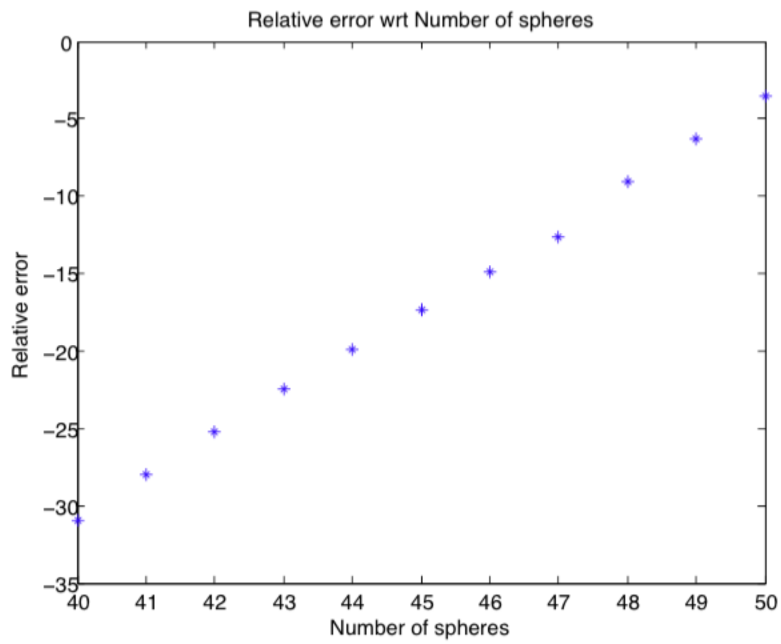

Figure 7. Relative error with respect to Number of spheres for Single Box method