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EFFECTIVE COULOMB FORCE MODELING IN A SPACE ENVIRONMENT

Laura A. Stiles,[*] Carl R. Seubert,[†] and Hanspeter Schaub[‡]

Coulomb formation flight is an emerging concept that utilizes electrostatic forces to maintain a formation of close proximity spacecraft. This paper uses analytic models and numerical simulations to explore the extent of plasma environment shielding on Coulomb forces with large potentials relative to the ambient plasma energy. The use of effective Debye lengths are used in analytic models to approximately and numerically efficiently calculate the force between charged objects. This is computed specifically for Coulomb free-flying formations and tethered Coulomb structures with nodal separations at dozens of meters operating in the geosynchronous plasma environment. It is shown that the force between a sphere and point charge is accurately captured with the effective Debye length, as opposed to the classic Debye length solutions that have errors exceeding 50%. One notable finding is that the effective Debye lengths in low earth orbit plasmas about a charged body are increased from the centimeter to meter level. This is a promising outcome, as the reduced shielding provides sufficient force levels for operating the electrostatically inflated membrane structures concept at these dense plasma altitudes.

INTRODUCTION

John Cover proposed the use of Coulomb forces in space in the 1960's as a means to inflate large-scale parabolic antennas.[1] Cover proposed the use of a charging source to inflate electrically conductive surfaces with a repulsive or attractive Coulomb force. By holding a charge, the reflector maintains its position relative to a radio frequency feed. It was proposed that a 30-40 foot diameter reflector at Geosynchronous Earth Orbit (GEO) requires potentials on the order of one to several tens of kilovolts and watt to kilowatt levels of power depending on the environment.[1]

More recently, the use of electrostatics in space is a progressive research area that encompasses many conceptual applications. In 2001, King and Parker proposed the use of Coulomb forces to control the relative dynamics of a free-flying formation of spacecraft.[2] Their study concluded that is is feasible to operate a 20-30 m size array for interferometry from GEO and the concept warrants further analysis. Building on this theoretical work, static equilibrium configurations are examined,[3–5] as well as the development of algorithms to control these naturally unstable formations.[6–10] An illustration of a simple two spacecraft formation using Coulomb forces for separation distance control is shown in Figure 1.

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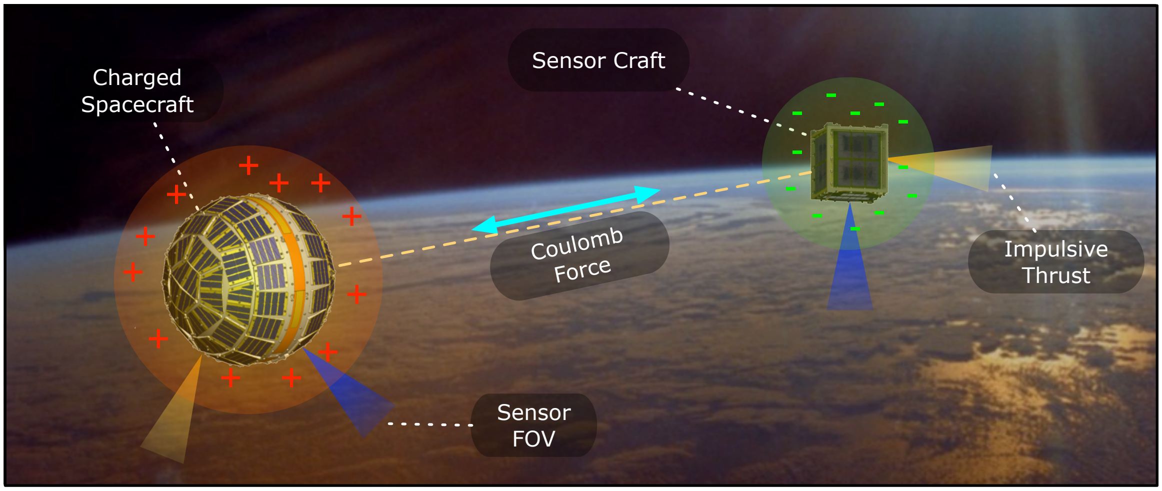


Figure 1. Two-craft Coulomb formation flight concept; active charge emission is used to charge craft to kilovolt-level potentials to control the separation distance with electrostatic forces

The use of Coulomb forces in "tractor" applications to manipulate the orbit of an object is also investigated. Schaub and Moorer present a concept that uses electrostatic forces to tug a GEO debris body.[11] In this scenario, the tug craft uses conventional thrusters to re-orbit the formation that is under electrostatic attraction without needing inter-spacecraft contact. In a similar application, Murdoch et al. propose the "electrostatic tractor" to deflect Near Earth Objects (NEO).[12]

The techniques of Coulomb formation control led to the development of the Tethered Coulomb Structure (TCS) concept.[13,14] Combining the features of large space structures and free-flying formations, a TCS uses Coulomb forces to repel a formation of spacecraft nodes that are connected with fine, low-mass tethers, creating large quasi-rigid and lightweight space structures. Most recently there is the proposed Electrostatic Inflated Membrane Structures (EIMS) concept for inflation and stiffening of gossamer structures.[15] Utilizing connected, lightweight conductive membranes, Coulomb forces inflate the structure for applications such as drag de-orbiting or radiation shielding.

A common requirement of these theoretical studies and applications is a model of the Coulomb force. The interaction between charged bodies and their electric fields (E-fields) and the resulting Coulomb force can be complex even for simple spherical shapes. Modeling this Coulomb force is further complicated with the interactions of the variable plasma environment or the inclusion of multiple bodies, non-spherical objects and attitude dependencies. For detailed modeling finite element, numerical electrostatic solvers are typically used. While these solvers offer an accurate solution, they can require significant processing power and time.

For simplicity in theoretical developments, analytic models are often desired and used. A review of these analytic Coulomb force models is presented in this paper, and an adapted model for close proximity craft in Earth orbit plasmas is proposed. The Coulomb force between two point charges in a vacuum, based on Laplace potential fields, is used and has validity in certain applications where the plasma charge shielding properties are negligible, such as nominal geostationary space weather conditions.[10,16] However for finite shapes in a plasma the electrostatic force is partially screened by the free flying particles In this scenario the vacuum model over over predicts the electrostatic force magnitude. A common practice to account for the partial charge shielding in a plasma is to use the conservative Debye-Hückel force model. This analytic representation has been used by King, Izzo, Saaj, Lappas, Peck, and Schaub.[2,8,9,17,18] These studies use point charges and do not consider the variations in system capacitance from having finite bodies in close proximity in a plasma. It is demonstrated here that this analytic approximation gives a conservative lower bound of the Coulomb force between points in a plasma. An analytic representation of the force between finite bodies in a plasma based on this Debye-Hückel force model is investigated in this paper.

The true Coulomb force between charges in a plasma is bounded by the vacuum and Debye-Hückel models. An alternative method to more accurately capture the force between charged spheres is suggested by Murdoch et. al. in Reference.[12] Murdoch proposes the use of an effective Debye length to use in a Debye-Hückel force model. This effective Debye length, which is longer and consequently reduces the extent of force shielding, is computed from numerically fitted solutions. The Murdoch application is for deflection of NEO asteroids, hundreds of meters in size in deep space. In this paper, an effective Debye length approach is pursued to study the plasma-shielded electrostatic forces for smaller meter-level bodies operating in Earth orbit plasmas. Of interest is how stronger Debye shielding conditions, such as scenarios where the Debye lengths are on the order of the spacecraft radii, impact the concept of Coulomb actuation of man-made spacecraft or membrane structures.

This paper provides an overview of the analytic force modeling techniques typically used and proposes a analytic representation of the force between two finite spheres in a plasma. This analytic approximation includes the variation in system capacitance due to both the close proximity spheres and the plasma interaction. Of importance here is the computation of effective Debye lengths in Earth orbit plasmas and the resulting charge production for meter-size craft and inflatable structures. Effective Debye lengths are quantified for GEO as well as Low Earth Orbit (LEO). Numerical solutions are used to compute the effective Debye lengths for these regimes and then used to verify the analytic force between a sphere and point charge.

The goal is to model the Coulomb force between spacecraft operating in Earth orbit plasmas. This includes quantifying the force capabilities for the Coulomb Formation Flight (CFF) concept and extensions to Coulomb applications such as the TCS and EIMS. This is intended for spacecraft operating in close separations (dozens of meters) as well as membrane and actuation devices operating on the cm level separations. Effective Debye lengths are computed for meter-size craft operating at tens of kilovolts in both LEO and GEO plasma conditions. The effect of these effective Debye lengths on the total charge of the craft and the resulting Coulomb force magnitude is explored.

GEO AND LEO PLASMAS & DEBYE LENGTHS

The Coulomb spacecraft applications require the use of a charge control devices to maintain a desired potential. This is achieved with an ion or electron emitter. Spacecraft will also naturally charge due to the interaction with the local plasma environment. Orbital missions such as SCATHA and ATS-6 were designed and launched specifically to characterize and quantify the extent of natural spacecraft charging.[19–21] On-orbit studies such as these have established that a GEO spacecraft can naturally charge to kilovolt-level potentials.[22,23]

There have also been a number of spacecraft that have studied the plasmas of the Earth orbit regimes. The plasma data used in subsequent sections is obtained from on-orbit measurements from the Spacecraft Charging AT High Altitudes (SCATHA) and the Applications Technology Spacecraft (ATS-5 & ATS-6), spacecraft during the 1970-80s. Interpretation of this data comes from Purvis,[24] Tribble,[25] and Pisacane.[26] There is also data obtained from the Magnetospheric Plasma Analyzer (MPA) instruments onboard the Los Alamos National Laboratory (LANL) spacecraft. These spacecraft operate at a range of longitudinal locations around the GEO belt.

In order to quantify Coulomb force magnitudes and the extent of plasma partial shielding, it is necessary to have a representative model of the GEO plasma environment. This section details the representations of the Earth orbit plasma (densities and temperatures) and the corresponding Debye lengths. Although offering simple insight, it is difficult to model plasma environments with nominal density and velocity values (Maxwellian distributions) as conditions can vary rapidly and with large fluctuations.[27] The local plasma conditions depend on the local time as well as solar interactions with the geomagnetic activity.[28]

Table 1 lists the plasma temperature and densities that are used in this study. The GEO values are based on References 29 and 30 that use data from the ATS-5 and ATS-6 spacecraft respectively. The LEO data is obtained from Reference.[2] Two representative GEO plasma conditions are used for this analysis (quiet and nominal). The quiet is an extreme bound at GEO that represents the "worst-case" conditions and highest force shielding. Nominal plasma conditions are a closer representation of the typical operating conditions at GEO. It is common to define the plasma with a thermal energy in units of eV. In this paper the temperature ($\mathcal{T}_e$) in eV is converted from Kelvin using $\mathcal{T}_e = \kappa T_e/e_c$.

Table 1. Representative GEO & LEO single Maxwellian plasma parameters and Debye lengths

Conditions	$\mathcal{T}_e$ [eV]	n_e [cm^{-3}]	λ_D [m]
LEO Nominal	0.2	1×10^5	**0.01**
GEO Quiet	3	10	**4**
GEO Nominal	900	1.25	**200**

The Debye length (λ_D) is a quantitative measure of the temperature and density of the local plasma.[26] It is a dimensional scale that parameterizes screening of the electric fields (E-fields) of a charged body. The sphere of influence of a charged body is defined by the Debye sphere that has a radius of one Debye length. Inside of the Debye sphere the free-flying plasma particles are influenced by the charged body E-fields and screen the potential field so that outside the Debye sphere the charge is effectively shielded. The plasma Debye sheath (δ_D) is the region beyond which charged bodies have no influence on the plasma. This can be multiple Debye lengths.

At GEO the plasma has Debye lengths ranging from 4 to 1000 m with a nominal value of approximately 200 m .[2,13] Debye lengths of this scale allow the use of Coulomb repulsion when operating with spacecraft separations of dozens of meters at GEO. The LEO Debye lengths are typically at the cm level and the interplanetary medium is typically at the meter-level.[12]

Although the plasma is a neutral mix of electrons and ions the Debye length is computed using solely the electrons, neglecting the influence of the more massive ions. This representation is referred to the "classical" or unperturbed plasma Debye length in this paper and is computed using:

$$\lambda_D = \sqrt{\frac{\epsilon_0 \kappa T_e}{n_e e_c^2}} \tag{1}$$

where $\epsilon_0 = 8.854187817 \times 10^{-12}$ $C^2N^{-1}m^{-2}$ is the permittivity of vacuum, $\kappa = 1.38065 \times 10^{-23}$ JK^{-1} is the Boltzmann constant, and $e_c = 1.602176 \times 10^{-19}$ C is the elementary charge. This is a suitable when the timescales of the process are short relative to the mobility of the ions.[31]

For a body charged to kilovolt-level potentials, the local plasma within the Debye sphere is perturbed. To incorporate the effects of charged bodies on their local plasma, an effective Debye length ($\bar{\lambda}_D$) is used. This effective Debye length is linearly proportional to the Debye length using a scaling parameter α using the relationship:

$$\bar{\lambda}_D = \alpha \lambda_D \tag{2}$$

The effective Debye length is computed with numerical solutions and is a function of parameters such as potential, plasma and craft size. The benefit of the approach using this effective Debye length is that it allows efficient analytic force computations with improved accuracy.

Murdoch in Reference[12] computes effective Debye lengths for analyzing the capabilities of for orbit altering and impact deflection of a NEO asteroid. Their study indicates, with particular examples, that the Coulomb application works best for 100 m size NEO, charges of 20 kV and mission durations up to 20 years. In this NEO application the interplanetary Debye length is 7.4 m, however with potentials up to 20 kV, Murdoch calculates that the effective Debye lengths can be as great as 349 m.[12] This is a scaling increase of approximately 50, and can result in significantly less plasma partial shielding of the Coulomb forces.

This effective Debye length study by Murdoch is used as a basis here to analyze the force production in a plasma for small spacecraft, operating in close proximities in Earth orbit plasmas. For charged craft in the dense plasma of LEO it is shown that the Debye lengths are scaled to the meter level. It is demonstrated that this improves the Coulomb forces magnitudes and makes them viable for applications such as inflation of membranes at cm level separations.

COULOMB FORCE MODELING IN A VACUUM

This section provides a dense overview of the Coulomb force models that are to be utilized for the CFF, TCS or EIMS applications. This section assumes that the plasma environment has a negligible impact on the electrostatic force evaluation. The force between point charges is expanded to that between finite spherical bodies to use as a basis for the plasma electrostatic force approximation developments in the following section.

The electrostatic force between two infinitesimally small point charges q_A and q_B is computed with the well known Coulomb's Law:

$$\boldsymbol{F} = k_c \frac{|q_A q_B|}{d^2} \hat{d} \tag{3}$$

where $k_c = 8.99 \times 10^9$ $\mathrm{Nm^2C^{-2}}$ is the Coulomb constant, and d is the radial distance between the point charges. It is an equal and opposite force acting on each point charge.

Force between sphere and point charge

Consider now that charge A is a finite sphere of radius R_A. In a vacuum without neighboring charged objects the potential on the surface of this isolated sphere is represented by the equation:

$$V_A = k_c \frac{q_A}{R_A} \tag{4}$$

At a radial distance from the center of this sphere ($r \geq R_A$), the potential field strength that radiates isotropically from this isolated charge is computed with:

$$\Phi(r) = k_c \frac{q_A}{r} = \frac{V_A R_A}{r} \tag{5}$$

The E-field strength of this charge is then:

$$E(r) = -\nabla_r \Phi(r) = \frac{\Phi(r)}{r} = k_c \frac{q_A}{r^2} = \frac{V_A R_A}{r^2} \tag{6}$$

If an infinitesimally small point charge, q_B is placed in this E-field at a distance d, the Coulomb force magnitude felt by both the point charge and the sphere is:

$$F = E(r = d) \cdot q_B = k_c \frac{q_A q_B}{d^2} = \frac{V_A R_A}{d^2} q_B \tag{7}$$

The infinitesimal charge q_B has no effect on the overall charge on the sphere q_A, except that a force is exerted.

For the Coulomb formation flight concept development it is assumed that the potential of the bodies, not the charge, is directly controlled to a desired level. It is envisioned that the craft will have a conductive outer material with an equipotential surface. From an application standpoint it is advantageous to control the potential as it is more readily measurable than the entire charge of the body. The force produced between the two finite bodies is computed based on the total charge of the bodies, therefore it is advantageous to model this force. The next section computes the force between two finite spheres.

Force between finite spheres

Consider now two charged bodies with finite dimensions in close proximity. The overlapping potential fields will raise or lower the effective potential of each body and consequently the Coulomb force between them. This can be significant at kilovolt level potentials when the center-to-center separation is low relative to the sphere radii (separations less than approximately 10 sphere radii, $r < 10R$). The net potential of the spheres is computed by combining Equations (4) and (5) to produce the set of equations in matrix form:

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = k_c \begin{bmatrix} 1/R_A & 1/d \\ 1/d & 1/R_B \end{bmatrix} \begin{bmatrix} q_A \\ q_B \end{bmatrix} \tag{8}$$

where d is the is the center to center separation of the bodies. Given that the potentials V_A and V_B of the bodies are controlled, then this equation is inverted to yield the resulting net charges on each body:

$$\begin{bmatrix} q_A \\ q_B \end{bmatrix} = \underbrace{\frac{d}{k_c(d^2 - R_A R_B)} \begin{bmatrix} dR_A & -R_A R_B \\ -R_A R_B & dR_B \end{bmatrix}}_{C_{\text{V}}} \begin{bmatrix} V_A \\ V_B \end{bmatrix} \tag{9}$$

Here C_{V} is the capacitance of the combined charged system in a vacuum. This set of equations is expandable to N number of charged bodies of both positive and negative potentials. The charge solution of these equations is then used in Eq. (3) to compute the Coulomb force between the spheres with surface potentials V_A and V_B.

COULOMB FORCE MODELING IN A PLASMA

The resulting force between two charges in a plasma is manipulated by the free flying charged particles. The objective here is to use the vacuum force developments to explore analytic plasma E-force modeling.

Electric fields from a sphere

For a charged body in a plasma, the free-flying charged particles shield the potential field causing it to drop off more rapidly than the vacuum expression of Eq. (5). The properties of a plasma surrounding a charged body are governed by the Vlasov-Poisson series of equations. These second order partial differential equations cannot be solved analytically for the potential field about even a simple point charge in a plasma. Numerical solutions can be employed with techniques such as the turning point method.[32] However, if the body has a low potential compared to the local plasma thermal energy

$$e_c V \ll \kappa T_e$$

then a first order solution to the Taylor series expansion can be used to obtain the Debye-Hückel approximation of the craft potential field:[33,34]

$$\Phi(r) = \frac{V_A R_A}{r} e^{-(r-R_A)/\lambda_D} \tag{10}$$

The advantage of using this Debye-Hückel potential field is that it provides a simplified analytic solution without the need for numerically solving the full Poisson-Vlasov equations. The consequence of neglecting the higher order terms in the Poisson's partial differential equations is that the plasma shielding of the electrostatic fields is not as drastic. Thus, this is a conservative estimate on the potential function that might actually exist about the charged body in a plasma.[12]

Figure 2 demonstrates graphically the differences in the potential field from the surface of an isolated 1 m sphere charged to a potential of 50 kV between the vacuum and Debye-Hückel models. The vacuum potential field bounds the upper limit of the potential curve, while the Debye-Hückel lower limit is computed for a worst-case, quiet plasma, Debye length λ_D = 4 m. The true potential field decay will lie in the shaded region between these curves. As the Debye length increases the shaded area is reduced as the lines converge.

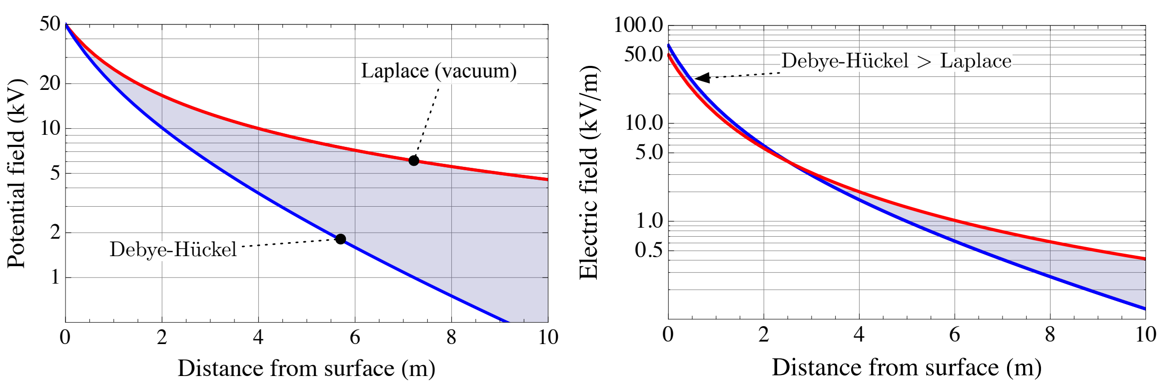


Figure 2. Potential and electric fields from an isolated, 1 m sphere diameter charged to 50 kV (quiet GEO plasma, λ_D = 4 m, used for the Debye-Hückel model

Taking the gradient of the potential function of Eq. (10) yields the spherically symmetric E-field for $r \geq R_A$:

$$E(r) = -\nabla_r \Phi(r) = \frac{V_A R_A}{r^2} e^{-(r-R_A)/\lambda_D} \left(1 + \frac{r}{\lambda_D}\right) \tag{11}$$

The E-field of a charged body in a plasma is also bound by the limits of this Debye-Hückel and Laplace fields, which are also shown in Figure 2. Due to the gradient of the potential function being larger at very close separations, the E-field for the Debye-Hückel model is actually larger than the Laplace, consequently the force in this region can also be larger. For the CFF concept this is of importance for deployment or docking conditions. Further, this plasma enhanced capacitance may be advantageous for close proximity Coulomb concepts such as the membrane structure developments. For CFF developments it is suitable for fundamental studies to use the analytic Debye-Hückel potential model in Eq. (10) as it provides a conservative lower limit of the resulting force production in a plasma.

Force between sphere and point charge

An analytic expression of the force between the sphere and point charge using the Debye-Hückel potential is developed. First, it is necessary to compute the charge of the sphere that maintains a desired surface potential V_A. Even for an isolated sphere, the plasma alters its capacitance so that the relationship between charge and potential, of Eq. (4), is altered. Assuming a homogenous surface charge density σ across the sphere (suitable given an isolated sphere and a well-mixed, neutral plasma), the total charge q residing on the surface is calculated with:

$$E(r = R_A) = \frac{\sigma}{\epsilon_o} = \frac{q}{A\epsilon_o} \tag{12}$$

Defining $A = 4\pi R^2$ as the spherical surface area and $k_c = 1/(4\pi\epsilon_0)$ as the Coulomb constant, the total charge on sphere A is estimated as:[34]

$$q_A = V_A \frac{R_A}{k_c} \left(1 + \frac{R_A}{\lambda_D}\right) \tag{13}$$

The resulting capacitance of an isolated sphere in a plasma is:[34, 35]

$$C_{\text{S}} = \frac{R_A}{k_c} \left(1 + \frac{R_A}{\lambda_D}\right) \tag{14}$$

This indicates that a craft that maintains a fixed potential will hold a charge that depends on the local plasma. If the plasma Debye length is very small (i.e. LEO regime), the space weather could have a significant impact on the sphere's capacitance, and its effective charge. If this plasma has minimal interaction (large Debye lengths, $R_A \ll \lambda_D$) this charge on the isolated sphere reduces to the classical vacuum formulation of Eq. (4). If the second charge is an infinitesimal point charge that does not effect the charge of the sphere the resulting Coulomb force is computed based on Eq. (11) with:

$$F = \frac{V_A R_A q_B}{d^2} e^{-(d-R_A)/\lambda_D} \left(1 + \frac{d}{\lambda_D}\right) \tag{15}$$

Simplified analytic force between finite spheres

For CFF studies it is advantageous to have a model of the force between two finite spheres in a plasma. Presented here is a simplified analytic expression that is derived in a similar manner to the earlier vacuum case. Rearranging Eq. (13) gives the charge on a sphere due to its potential, including the capacitance from the local plasma. If there are two bodies in close proximity it is also necessary to include the effects of the second sphere. Combining Eqs. (10) and (13) yields:

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = k_c \underbrace{\begin{bmatrix} \frac{1}{R_A}\left(\frac{\lambda_D}{R_A+\lambda_D}\right) & \frac{1}{r}\left(\frac{\lambda_D}{R_B+\lambda_D}\right) e^{\frac{-(r-R_B)}{\lambda_D}} \\ \frac{1}{r}\left(\frac{\lambda_D}{R_A+\lambda_D}\right) e^{\frac{-(r-R_A)}{\lambda_D}} & \frac{1}{R_B}\left(\frac{\lambda_D}{R_B+\lambda_D}\right) \end{bmatrix}}_{C_{\mathrm{P}}^{-1}} \begin{bmatrix} \bar{q}_A \\ \bar{q}_B \end{bmatrix} \tag{16}$$

where C_{P} is the capacitance of the combined charged system including the plasma. For a desired surface potential on each sphere this system of equations are inverted to solve for the equivalent charges. These charges are used to compute the resulting Coulomb force between the spheres using:

$$F = k_c \frac{\bar{q}_A \bar{q}_B}{r^2} e^{-(r-R_A)/\lambda_D} \left(1 + \frac{r}{\lambda_D}\right) \tag{17}$$

which is obtained from Eq. (11) by substituting for V_A using the vacuum capacitance relationship of Eq. (4). The effects of the plasma on capacitance are still captured when solving for the charge with Eq. (16). Again, this is a conservative estimate of the force between charged bodies in a plasma and is best suited when the low spacecraft potential $eV \ll kT$ is valid.

Violation of ($eV \ll kT$) Assumption

The Debye-Hückel potential field and resulting Coulomb force model is an analytic expression that is derived by assuming $eV \ll kT$ holds. Table 1 quantifies the spacecraft surface potential required to match the plasma thermal energy, i.e. $eV = kT$. For all plasma conditions except the rare disturbed environment the potentials required are well below the kilovolt levels that are required for the Coulomb formation flight concept.

Table 2. Craft voltages violating the ($eV \ll kT$) assumption

Plasma conditions	Debye length [m]	Craft potential [V]
LEO Nominal	0.01	0.2
GEO Quiet	4	3
GEO Nominal	200	900

If the craft potential is much less than plasma energy ($eV \ll kT$), than the Debye-Hückel potential of Eq. (11) is a good approximation. If the craft potential is significantly greater than the plasma ($eV \gg kT$) than the plasma-based potential field is closer to the vacuum model of Eq. (6). For the Coulomb formation flight application the potentials and plasma properties have similar magnitudes. Consequently the two approximations available provide bounds on the range of potential decay from a charged body in a plasma. The resulting Coulomb force that is derived from these potential fields is also bound by these analytic representations. One method to analytically compute the force within this range with higher accuracy is with the effective Debye length, as proposed by Reference 12 for the charged asteroid scenario. The suitability of the effective Debye length for partially plasma shielded E-Force evaluations of man-made spacecraft is investigated in the this section.

EFFECTIVE DEBYE LENGTHS AND COULOMB FORCES

For Coulomb applications in GEO and LEO orbit regimes, the $(eV \ll kT)$ assumption in the analytical development of the Debye-Hückel potential is quickly violated. The Debye-Hückel potential of Eq. (10) serves as a lower bound for the actual potential decay from a charged body in a plasma while the the Laplace potential of Eq. (5) serves as an upper bound. One method to analytically compute the force within this range with higher accuracy is with an effective Debye length. This effective Debye length is larger than the true Debye length and consequently reduces the screening of the potential field. It can then be substituted directly into the Debye-Hückel Coulomb force model to more appropriately match the true force.

The effective Debye length, given in Eq. (2), is numerically computed based on plasma conditions, craft size, and potential as demonstrated in Reference 12. In this paper, the effective Debye lengths are obtained from numerical solutions of the Poisson-Vlasov equation in the OML (Orbit Motion Limited) limit. Solutions are obtained for the E-field surrounding a charged sphere in a plasma. An α scalar value described by Eq. (**??**) is determined by fitting an effective Debye shielded E-field model to the numerical solution across distances up to several Debye lengths from the sphere. The E-field model used is based on Eq. (11), using effective Debye lengths:

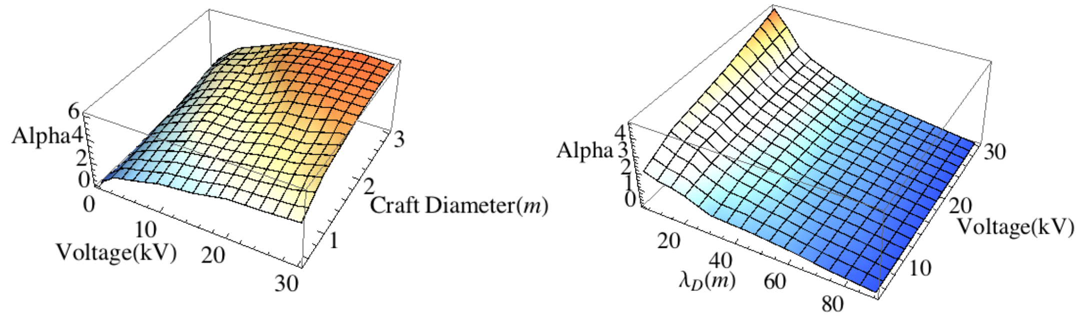
$$E(r) = -\nabla_r \Phi(r) = \frac{V_A R_A}{r^2} e^{-(r-R_A)/\alpha\lambda_D} \left(1 + \frac{r}{\alpha\lambda_D}\right) \tag{18}$$

The effective Debye length is computed for force computations in a plasma specifically for the Coulomb formation flight concept in Earth plasmas for a range of craft sizes and craft potentials. The nominal GEO plasma conditions are not investigated as the plasma shielding in a Debye length of 200 m is minimal and the Debye-Hückel model closely resembles the vacuum values.

Effective Debye Length Trends Factors which affect the effective Debye length include the plasma Debye length, craft radius, and the voltage level on the craft. Numerical simulations were performed using ranges of each of these parameters to understand the trends in α-parameter. Figure 3(a) displays the variation of the α factor over a range of craft sizes and craft voltages.

Figure 3(a) clearly illustrates that larger craft radii and higher potentials yield larger effective Debye lengths. The craft voltage trend occurs because as the assumption $(e\phi \ll kT)$ is further violated, the shielding is reduced. There exist limitations on craft voltage due to power requirements and limitations on craft size due to launch constraints, but this study demonstrates that electrostatic forces are more effective with larger craft and higher potentials.

Figure 3(b) shows the variation of the effective shielding parameter as a function of Debye length and craft voltage for a sphere with a 1m diameter. The extreme case of a quiet GEO plasma Debye length near 4 m and a high craft voltage of 30 kV, the effective Debye length is more than 5 times the predicted Debye length. As the Debye length approaches 100 m, the effective Debye length is close to the predicted Debye-Hückel Debye length, therefore α approaches a value of one. With a Debye length of 100 m, the Debye shielded force is nearly the same as the vacuum force at distances



(a) Craft diameter and potential in a fixed quiet plasma ($\lambda_D = 4$)

(b) Craft potential and Debye length for a fixed diameter of 1 m

Figure 3. Trends in Alpha parameter for effective Debye lengths in a GEO plasma

on the order of a few tens of meters. Interpolation of the data shown on these plots can be used to determine the effective Debye lengths for different combinations of craft voltage, Debye lengths, and craft sizes.

GEO Effective Debye lengths

In this section, E-fields surrounding a charged craft in a quiet GEO plasma are examined in more detail. Figure 4 compares the vacuum E-field, the analytical E-field model using both classic Debye length and the effective Debye length, and the numerical simulation results. Two cases are illustrated: craft voltage of 1kV (top) and a craft voltage of 30kV (bottom). In both cases, the numerical solution has a stronger E-field than classically predicted, but have an upper bound of the Laplace potential of Eq. (5) (vacuum case). As the craft voltage increases, the actual E-field values stray further from the the classic Debye-Hückel model and approach the vacuum E-field.

Parametric α-Parameter Relationship For Coulomb force modeling, a parametric relationship between the effective Debye length α-parameter and input parameters of craft voltage and craft diameter is desired. This allows for quick modeling of the E-fields or forces surrounding the craft with improved accuracy from classic Debye length solutions.

A nonlinear model in the form of $V \cdot poly(V_C, D_C) + (1 + e^V) \cdot poly(V_C, D_C)$ was fit to α-parameter values across a a range of craft voltages sizes for the quiet GEO Debye. The coefficient of determination, $\mathcal{R}^2$, was used to measure the goodness of fit for the model. A nonlinear fit of the alpha parameter data is shown in Eq. (**??**) as a function of craft voltage, V_C and craft diameter, D_C. The $\mathcal{R}^2$ value of this fit is 0.995, very near the maximum value of 1, therefore this model was considered a satisfactory prediction of the alpha parameter for effective Debye lengths. The regression equation is as follows:

$$\begin{aligned}\alpha = 1 + 0.490V_C - 0.00322V_C^2 + 0.00446V_C D_C + \\ \left(1 - e^{-0.576V_C}\right)\left(-0.1045 - 0.289V_C + 1.086D_C\right)\end{aligned} \tag{19}$$

This nonlinear model forces the α parameter to converge to a value of 1 as the craft voltage approaches zero. As the voltage approaches zero, the $(e\phi \ll kT)$ rule is no longer violated, thus

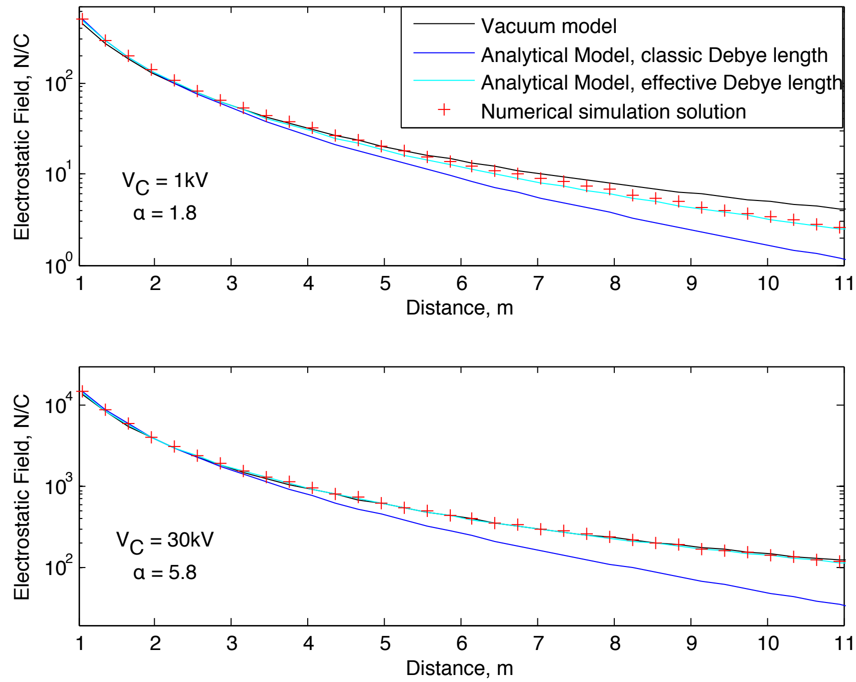


Figure 4. Comparison of classic E-field models with the effective Debye model; GEO plamsa, $\lambda_D = 4m$

the classic Debye-Hückel model is satisfactory for predicting E-fields and forces. The form of this nonlinear model was chosen as a combination of a quadratic polynomial term, which well models the behaviors at higher voltages (5 - 30 kV), and an exponential term which models the low voltage behavior as α converges to one. This equation is beneficial as quick evaluation of how much the Debye length should be modified to account for the $(e\phi \ll kT)$ violation when considering Coulomb spacecraft applications.

Illustration of Results To illustrate the results of the parametric relationship of Equation (19), a surface plot is shown in Figure 5. This plot displays the alpha parameter data as a colored surface, with the fit model shown as a mesh surface. In this figure, it can be seen that the model closely matches the behavior of the data.

LEO Effective Debye lengths

In the same procedure as described for determining the GEO effective Debye length, effective Debye lengths at LEO are investigated. As seen in Table 1, the nominal LEO Debye length is on the order of one centimeter. This small distance limits the feasibility of electrostatic actuation for Coulomb formation flying or tethered Coulomb applications. Other applications of electrostatics in LEO may however be feasible, for example electrostatically inflated membrane structures. These structures use electrostatic forces between layers of conducting membranes as the source of inflation pressure. The distances over which the electrostatic force acts is much smaller, likely on the order of a few centimeters. As shown in Table 2, $eV = kT$ at a craft voltage of 0.2V, therefore any Coulomb application with kilovolts of potentials will clearly violate the Debye-Hückel assumption. It is important to understand if the violation of $ev \ll kT$ will aid in electrostatic inflation or other

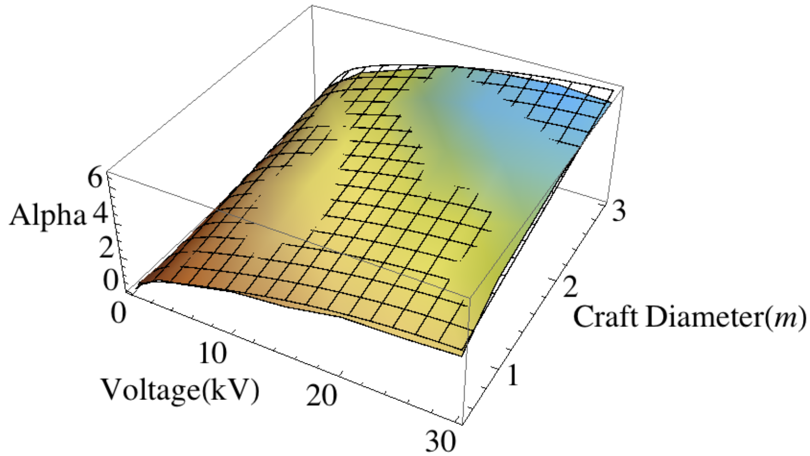


Figure 5. Alpha parameter values and surface plot of regression (Equation (19)**) for Quiet GEO plasma conditions, $\lambda_D = 4$; colored surface represents data and mesh represents fit surface**

small separation distance Coulomb applications.

Figure 6 illustrates the E-field models compared to numerical field results for a LEO plasma environment. The classic Debye length E-field model is not a good approximation of the numeric solution. As an improvement to the classic Debye length results, an effective Debye length analytical model was fit to the data for distances up to approximately two effective Debye lengths (α=24 for illustrated case). The scaling factors at LEO proved to be significantly larger than those at GEO, representing significantly less plasma partial shielding of the Coulomb forces. The LEO effective Debye lengths are on the order of several decimeters to a meter, as opposed to the centimeter level classic Debye lengths. This shielding reduction improves the Coulomb forces magnitudes and makes them viable for applications such as inflation of membranes at cm level separations.

The approach presented for modeling the effective Debye lengths provided good results at voltages in the range of 5kV to 30kV, the voltage range which would be employed for Coulomb applications. At lower voltages, the Debye-Hückel model could not be reasonably fit to the numerical solutions solely by using an effective Debye length. Other models would need to be considered for modeling voltages below 5 kV. The α values found in the 5kV to 30kV range are shown in Figure 7 for a range of crafti sizes and craft voltages. Interestingly, the α-parameter is nearly constant across the voltage levels, which was not the trend for the GEO case. This indicates that the effective Debye lengths converge to a limit as $eV < kT$ is strongly exceeded. As seen in the plot, the relationship across craft diameters is nearly linear. A linear model was fit to this data with an $\mathcal{R}^2$ value of 0.994. The expression for this linear model for calculating the α-parameter in this voltage range is:

$$\alpha = 7.028 - 0.031V_C + 42.314D_C \tag{20}$$

At LEO altitudes, the dense plasma can have interactions with the charged body that affect the potential. It is important to note that other plasma mechanisms, such as wake effects, are not being considered in this analysis.

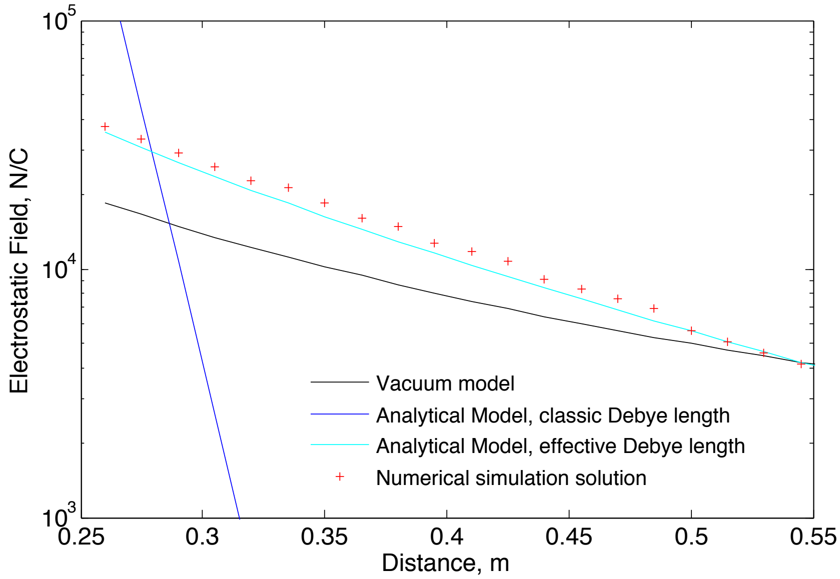


Figure 6. Comparison of classic E-field models with the effective Debye model; LEO plamsa, $\lambda_D = 0.011m$, $V_C = 5kV$

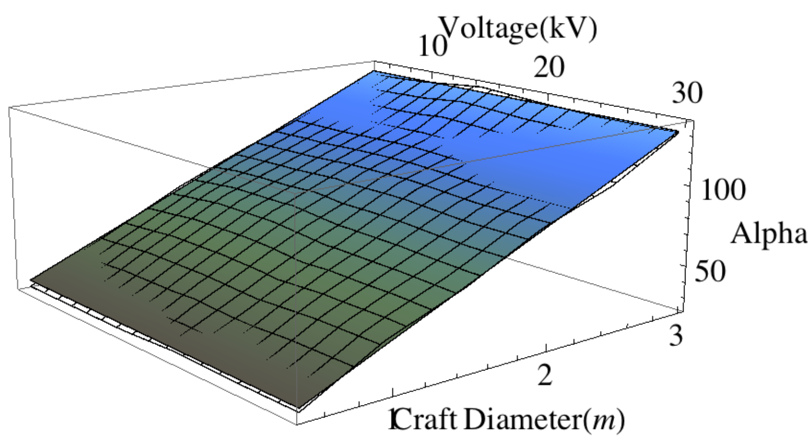


Figure 7. Alpha parameter values and surface plot of regression (Equation (20)) for LEO plasma conditions, $\lambda_D = 0.011$; colored surface represents data and mesh represents fit surface

VERIFICATION OF CHARGES AND FORCES

The numerical solver for the electric field about a sphere in a plasma is used to verify the accuracy of the proposed analytic models and use of the effective Debye length solutions. Both the charge computation of the isolated sphere in a plasma as well as the resulting force between the sphere and a point charge is compared.

Total charge on an isolated sphere in a plasma

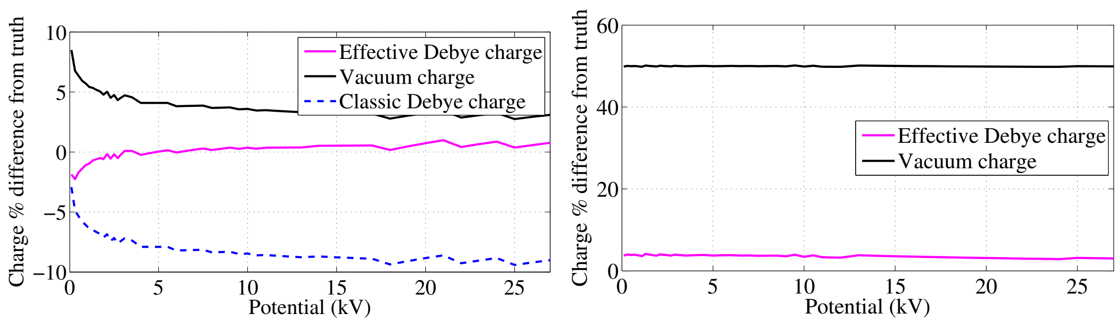
This analysis focuses on the quiet GEO plasma with a Debye length of 4 m as well as a LEO plasma condition with Debye length of 0.011 m. The charge on an isolated sphere is computed using the numerical solver and used as the truth value (q_{truth}). The 1-meter diameter sphere maintains a fixed potential. The charge is analytically computed using the vacuum expression in Eq. (4) and the plasma expression in Eq. (13) with the classic Debye length and also computed with the effective Debye length using:

$$q_A = V_A \frac{R_A}{k_c}\left(1 + \frac{R_A}{\bar{\lambda}_D}\right) \tag{21}$$

The percentage error of the charge computation from the numerical truth is calculated using

$$\% = 100 \times (q_{\text{truth}} - q_i)/q_{\text{truth}}$$

and shown in Figure 8 for each plasma condition.



(a) GEO quiet $\lambda_D = 4$ m

(b) LEO $\lambda_D = 0.11$ m

Figure 8. Computation of charge on an isolated sphere using analytic models compared to the numerical truth as a percentage difference over a range of CFF potentials

Figure 8(a) shows the percentage difference in the sphere's charge for the quiet GEO plasma. In this situation, the computation of the charge using the effective Debye length is $< 1\%$ across all potentials. The vacuum Debye length under estimates the charge magnitude, while the charge computed with the classic Debye length over estimates the charge of the sphere.

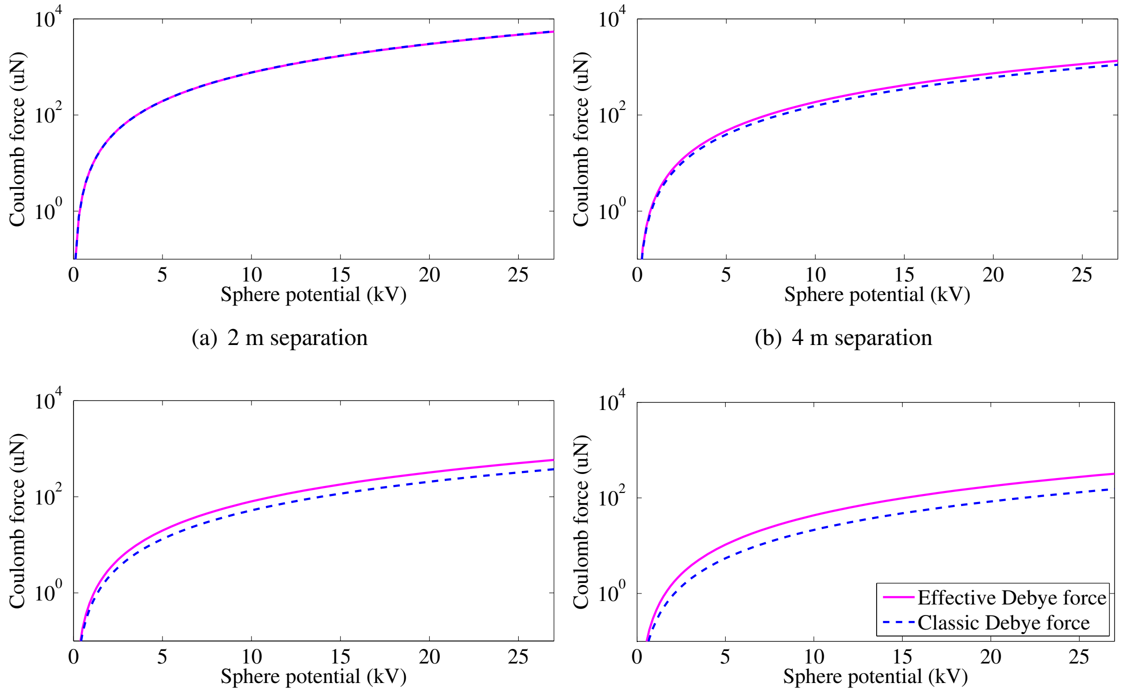
Figure 8(b) shows the percentage difference in the sphere's charge for the LEO plasma. In this case the effective Debye length computation is within 4% of the truth across all potentials. The vacuum computation under estimates the charge magnitude by $\approx 5\%$ and the classic Debye computation severely over estimates the charge with an error of $\approx -2000\%$ and is not shown in this scale.

This study shows the importance of using the effective Debye length to compute the self capacitance of a sphere in a plasma. It is necessary to use the effective Debye length in these and similar plasma environments as it allows accurate computation of the force between sphere and point charge as shown in the next section.

Forces

In this section the force between a sphere and a point charge in a quiet GEO plasma is computed with both the numerical solver and analytic models and compared. Figure 9 shows the absolute force levels as a function of potential for a 1 meter sphere with a point charge separated from the sphere center by 2, 4, 6, and 8 meters. The force is shown for the classic Debye length shielding using Eq. (15) as well as the effective Debye length computed using:

$$F = \frac{V_A R_A q_B}{d^2} e^{-(d-R_A)/\bar{\lambda}_D} \left(1 + \frac{d}{\bar{\lambda}_D}\right) \tag{22}$$



(a) 2 m separation

(b) 4 m separation

(c) 6 m separation

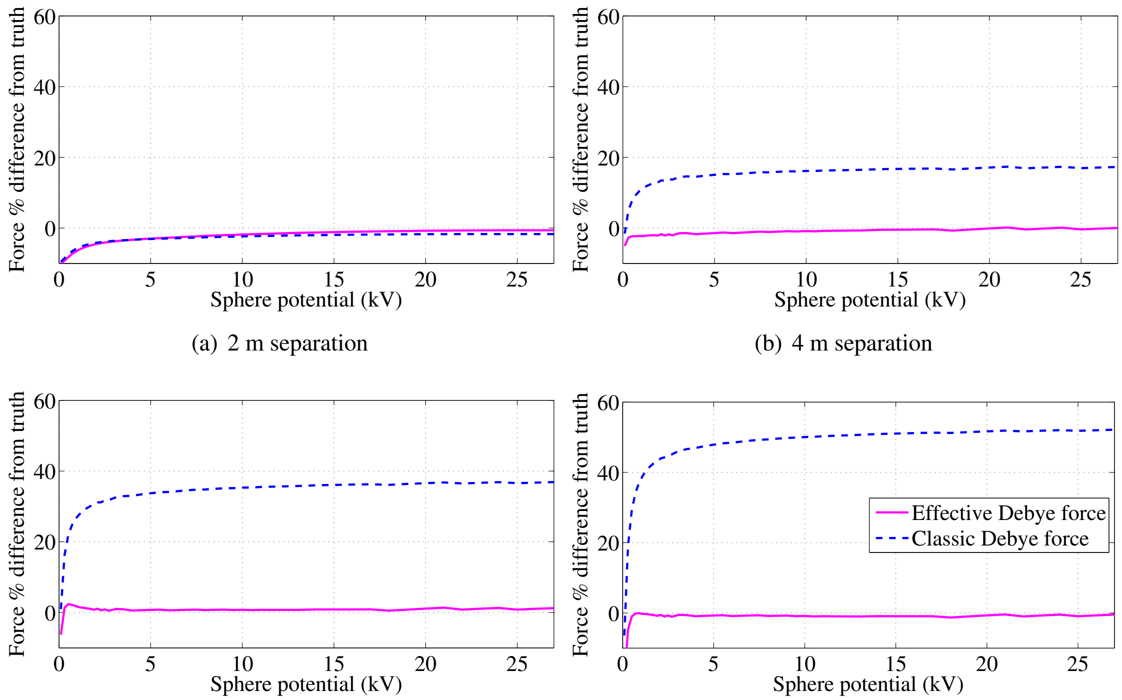
(d) 8 m separation

Figure 9. Force between a 1 m diameter sphere and point charge over a range of CFF potentials for four different separations in a quiet GEO plasma ($\lambda_D = 4$ m)

It is shown in Figure 9 that the force magnitude is greatest at the closest separation (2 m) and the analytic representations are very similar. As the separation increases, the force magnitude decreases and the effective Debye length solutions diverge from the classic Debye force. The consequence of this for CFF is that the classic Debye analytic force models used are an underestimate of the force magnitude in a dense plasma that can differ substantially at larger, yet realistic separations.

It is beneficial to quantify the accuracy of the analytic models to the numerical force solution. The percentage error of the force computation from the numerical truth is calculated using $\% = 100 \times (F_{\text{truth}} - F_i)/F_{\text{truth}}$ and shown in Figure 10 for each separation distance. This demonstrates that the force calculated with the effective Debye length model is within 5% of the numerical truth for all separations. The force calculated with the classical Debye length in the analytic model is accurate at the close separations but at separations of 8 m underestimates the force as much as

50%. This indicates that for the force between a sphere and point charge the analytic model using a generic alpha function (effective Debye length) accurately predicts the force magnitude in a GEO quiet plasma.



(a) 2 m separation

(b) 4 m separation

(c) 6 m separation

(d) 8 m separation

Figure 10. Comparison of analytic forces with classic and effective Debye lengths as a percentage difference from truth between a 1 m diameter sphere and point charge over a range of CFF potentials for four different separations in a quiet GEO plasma ($\lambda_D = 4$ m)

CONCLUSIONS

In this paper, the plasma environment effects on Coulomb force calculations for Coulomb spacecraft applications are explored. An analytical force calculation for two spheres is developed which includes plasma Debye shielding and accounts for finite bodies. This analytical calculation is based on the assumption that the body has a low potential compared to the local plasma thermal energy. This assumption, however, is quickly violated for charged craft in LEO and also in quiet GEO conditions. Numerical simulations allows for a more accurate solution to the forces and a modified 'effective' Debye length can be defined to allow quick use of the analytical equations. These effective Debye lengths are calculated for GEO and show that the effective Debye length can be from several times larger than the calculated Debye length for the applications of Coulomb formation flying. In LEO plasma conditions, the 'effective' Debye length can be more than an order of magnitude larger than the classically predicted Debye length. The LEO effective Debye lengths can therefore be up to the meter level, and the resulting Coulomb force improvement from reduced shielding may allow for LEO Coulomb spacecraft applications such as inflation of membranes at cm level separations.

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