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Inverted Pendulum Control via Variable Speed Control Moment Gyroscope

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Abstract: Variable speed control moment gyroscope (VSCMG) is an innovative aerospace actuator used to control the attitude of satellites and spacecraft. This kind of mechanism gathers the favorable aspects of using a traditional control moment gyroscope (CMG) but with the spin rate of the reaction wheel variable, being able to avoid singularities that occur when using CMGs. Thus, this paper presents the model of a non-usual pendulum configuration where a Variable Speed Control Moment Gyroscope is coupled to it. The idea of the VSCMG pendulum is to use the angular momentum variation in amplitude and direction using the system dynamics to perform the control. In this way, the controller was designed using Lyapunov control theory in order to control the pendulum in the inverted position. So, results are presented to illustrate how this new pendulum configuration was controlled.

Keywords: reaction wheel, pendulum, nonlinear control, nonlinear dynamics, Variable Speed Control Moment Gyroscope - VSCMG

INTRODUCTION

The attitude control in devices has been attractive due to a series of challenges because usually devices such as satellites and robots are described by nonlinear motion equations and several uncertainties related with their parameters and with high-level disturbances that occur during operation. The use of common space actuators in other fields can be more explored and in particular, the control and stabilization of mechanisms using reaction wheels have been common due to its simple configuration and its reliability to aerospace industry applications (Rui et al. 2000).

On the other hand, different configurations of mechanisms coupled to pendulums are used to approximate several real-world systems, like the reaction mass pendulum (Sanyal and Goswami 2014) that approximates the dynamics of a robot or the wheeled inverted pendulum (Vasudevan et al. 2015) that has a similar behavior than a Segway and many others. The control study of these systems has been done for a long period, always helping research development of controllers that can be tested in these simple mechanisms (Grasser et al. 2002; Sanyal and Goswami 2014; Vasudevan et al. 2015;). The use of simple models that behaves as an inverted pendulum has provided a variety of problems that helped the development of nonlinear dynamics and control.

In this way, this paper presents a new non-usual configuration of an inverted pendulum that has a Variable Speed Control Moment Gyroscope - VSCMG couple to it. Thus, the modeling of this new configuration is presented and a nonlinear controller is derived using Lyapunov control theory. Afterwards, the results obtained for taking the pendulum from the downward position to the inverted position are presented. Finally, the concluding remarks are stated.

MODEL OF THE VARIABLE SPEED CONTROL MOMENT GYROSCOPE PENDULUM

This section presents the mathematical modeling using Newton-Euler dynamics of the new pendulum configuration proposed: the VSCMG pendulum. In order to model the Variable Speed Control Moment Gyroscope coupled to a pendulum, two moving frames were used. The inertial frame is described by axis (x, y, z) and orthonormal basis $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$. The first moving frame (B), or body frame, is described by axis (x_1, y_1, z_1) and orthonormal basis $\{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$ is rotating solidarity to the pendulum with the angle θ considering a negative rotation. The second moving frame (G), or gimbal frame, is described by axis (x_2, y_2, z_2) and orthonormal basis $\{\hat{g}_t, \hat{g}_g, \hat{g}_s\}$. This frame is responsible for rotating the gimbal with angle γ which makes the reaction wheel to move outside the plane, and for rotating the gimbal it is considered a positive rotation. The illustration of the VSCMG pendulum can be seen in Fig. 1.

In this new system, there are three different angular velocities to be evaluated. The first one is the pendulum angular velocity ${}^{B}\boldsymbol{\omega}_{B/N} = \dot{\theta}\,\hat{b}_3$, there is the gimbal angular velocity ${}^{B}\boldsymbol{\omega}_{G/B} = \dot{\gamma}\hat{b}_2$ and the reaction wheel angular velocity ${}^{B}\boldsymbol{\omega}_{W/G} = \dot{\psi}\sin\gamma\hat{b}_1 + \dot{\psi}\cos\gamma\hat{b}_3$. In this way, the inertial angular wheel velocity is ${}^{B}\boldsymbol{\omega}_{W/N} = \dot{\psi}\cos\gamma\hat{b}_1 + \dot{\gamma}\hat{b}_2 + (\dot{\theta} + \dot{\psi}\cos\gamma)\,\hat{b}_3$. Before calculating the angular momentum it is necessary to evaluate the inertia matrix. This is done for the pendulum, for the gimbal and for the reaction wheel. The main inertia directions of the gimbal and of the reaction wheel are in a different frame then the pendulum. In this case, it is necessary to rotate these inertia matrix to the body frame B because it is in this frame that the model of the VSCMG pendulum is going to be described.

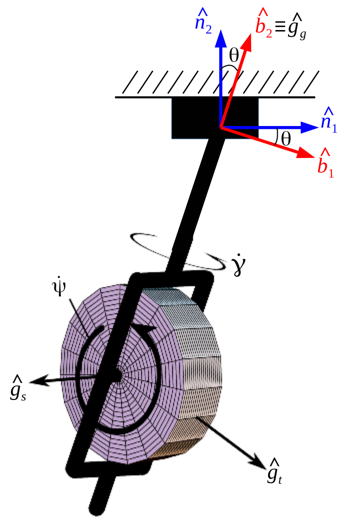


Figure 1 – Variable Speed Control Moment Gyroscope Pendulum

The total angular momentum of the system (${}^B\boldsymbol{H}$) is the sum of the angular momentum of the pendulum (${}^B\boldsymbol{H}_B$), of the gimbal (${}^B\boldsymbol{H}_G$) and of the reaction wheel (${}^B\boldsymbol{H}_W$) as shown below:

$$ {}^B\boldsymbol{H} = {}^B\boldsymbol{H}_B + {}^B\boldsymbol{H}_G + {}^B\boldsymbol{H}_W \tag{1} $$

that can be rewritten as:

$$ {}^B\boldsymbol{H} = {}^B[I_p]{}^B\boldsymbol{\omega}_{B/N} + {}^B[I_g]{}^B\boldsymbol{\omega}_{G/N} + {}^B[I_W]{}^B\boldsymbol{\omega}_{W/N} \tag{2} $$

where ${}^B\boldsymbol{\omega}_{G/N} = {}^B\boldsymbol{\omega}_{B/N} + {}^B\boldsymbol{\omega}_{G/B}$, ${}^B[I_p]$ is the inertia matrix of the pendulum, ${}^B[I_g]$ is the inertia matrix of the gimbal and ${}^B[I_W]$ is the inertia matrix of the reaction wheel. Now that the angular momentum was evaluated, the derivation of it can be calculated by:

$$ \dot{\boldsymbol{H}} \quad = \quad \frac{{}^B\mathrm{d}}{\mathrm{dt}}(\boldsymbol{H}) + {}^B\boldsymbol{\omega}_{B/N} \times {}^B\boldsymbol{H} = {}^B\boldsymbol{L} \tag{3} $$

This derivation is performed in order to find the equations of motion of the system. So, the equation of motion of the VSCMG pendulum obtained is:

$$ \mathscr{A}\ddot{\theta} + \mathscr{B}\ddot{\psi} + \mathscr{C}\dot{\gamma} + \mathscr{D} = 0 \tag{4} $$

where

$$ \mathscr{A} = I_{wt}\sin^2\gamma + I_{ws}\cos^2\gamma + I_p + I_{gt}\sin^2\gamma + I_{gs}\cos^2\gamma \tag{5} $$
$$ \mathscr{B} = I_{ws}\cos\gamma \tag{6} $$
$$ \mathscr{C} = \dot{\theta}\cos\gamma\sin\gamma(2I_{wt} + 2I_{gt} - 2I_{ws} - 2I_{gs}) - \dot{\psi}I_{ws}\sin\gamma \tag{7} $$
$$ \mathscr{D} = -rmg\sin\theta \tag{8} $$

In order to obtain the motor torque equation for the gimbal, it is necessary to analyze the gimbal separate from the whole system. The motor torque equation of the gimbal obtained is:

$$ (I_{gg} + I_{wt})\ddot{\gamma} + I_{ws}\dot{\psi}\dot{\theta}\sin\gamma = u_g \tag{9} $$

Equation (9) is the motor torque equation for the gimbal where u_g is the torque produced by the gimbal. It is also necessary to analyze the reaction wheel separately in order to obtain the motor torque equation for it. The following reaction wheel motor torque equation is obtained:

$$ I_{ws}\ddot{\theta}\cos\gamma - I_{ws}\dot{\theta}\dot{\gamma}\sin\gamma + I_{ws}\ddot{\psi} = u_s \tag{10} $$

where u_s is the torque provided by the reaction wheel. Now the model of the VSCMG pendulum is complete, where the equation of motion of the system was presented by Eq. (4), the gimbal motor torque equation in Eq. (9) and the reaction wheel motor torque equation in Eq. (10). The integration of this three equations simultaneously results in the motion of the VSCMG pendulum. For the VSCMG pendulum it has been considered a reaction wheel with radius $R = 0.11$ m, mass $m_w = 0.2151$ kg, a pendulum length of $\ell = 0.35$ m with a mass $m_p = 0.1$ kg. The gimbal structure has a mass $m_g = 0.082$ kg, and its inertia was obtained using *SolidWorks*® where $I_{gt} = 0.0020441$ kg.m^2, $I_{gg} = 0.0020466$ kg.m^2 and $I_{gs} = 0.0000054$ kg.m^2. The radius of the center of mass is $r = (m_g\ell/2 + m_g\ell + m_w\ell)/(m_p + m_g + m_w)$.

CONTROL DESIGN

This section presents how it has been designed the control law for the VSCMG pendulum. In order to design a feedback control law for the VSCMG pendulum using the Lyapunov control theory it is necessary to select a candidate to be a Lyapunov function:

$$\mathscr{V}(\delta\theta, \delta\dot{\theta}) = \frac{\delta\dot{\theta}^2}{2} + K\frac{\delta\theta^2}{2} \tag{11}$$

where $\delta\dot{\theta} = \dot{\theta} - \dot{\theta}_r$, $\delta\theta = \theta - \theta_r$ and K is a scalar attitude feedback gain. In order to the system be stable, it is necessary that the derivative of the Lyapunov candidate function be at least negative semidefinite. In this way, $\dot{\mathscr{V}}(\delta\theta, \delta\dot{\theta})$ is set to be equal to a negative definite function, in this case:

$$\dot{\mathscr{V}}(\delta\theta, \delta\dot{\theta}) = \delta\dot{\theta}\left(\delta\ddot{\theta} + K\delta\theta\right) = -P\delta\dot{\theta}^2 \tag{12}$$

By developing the derivative of the Lyapunov candidate function, it is possible to obtain the following stable closed-loop dynamical system:

$$\ddot{\theta} - \ddot{\theta}_r + P\delta\dot{\theta} + K\delta\theta = 0 \tag{13}$$

After obtaining the closed-loop system, it is necessary to substitute the equation of motion on it to obtain the control law. Analyzing the control law and calculating the higher order derivatives of the candidate for the Lyapunov function, it is possible to prove that the proposed control is asymptotically stabilizing. The second order derivative of the Lyapunov function candidate is $\ddot{\mathscr{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2P\delta\dot{\theta}\delta\ddot{\theta} = 0$. The third order derivative is given by $\dddot{\mathscr{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2P\delta\ddot{\theta}^2$ and from the closed-loop dynamics $\dddot{\mathscr{V}}(\delta\theta, \delta\dot{\theta} = 0) = -2P\delta(-K\delta\theta)^2 = -2PK^2\delta\theta^2 < 0$. The third derivative of $\mathscr{V}(\delta\theta, \delta\dot{\theta})$ is negative definite and because the first nonzero $\mathscr{V}$ derivative is of odd order, the control law is asymptotically stable. The control law obtained for the VSCMG pendulum is presented below:

$$[B \quad C]\left\{\begin{array}{c} \ddot{\psi} \\ \dot{\gamma} \end{array}\right\} = -d - A\ddot{\theta}_r + PA\delta\dot{\theta} + KA\delta\theta = L_r \tag{14}$$

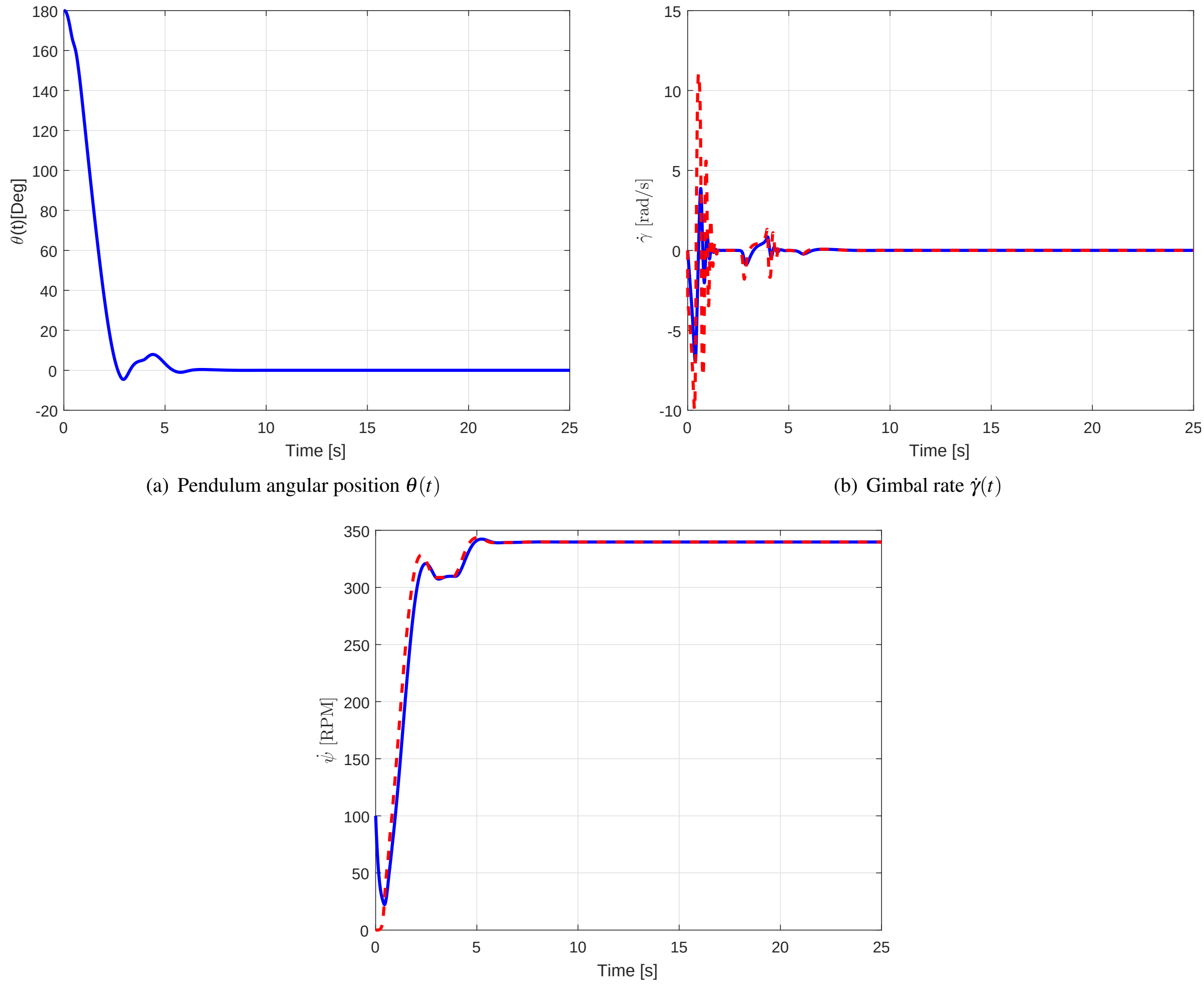
SIMULATED RESULTS

This section presents the results obtained to control the VSCMG pendulum in the inverted position. It has been assumed the following initial conditions for the simulations: $\theta(0) = 180°$, $\dot{\theta}(0) = 0$, $\psi(0) = 0$, $\dot{\psi}(0) = 100$ RPM, $\gamma(0) = 90°$ and $\dot{\gamma}(0) = 0$. For the controller designed, the gains used in the simulations were $K = 5$ and $P = 5$.

The goal of this paper is to control the VSCMG pendulum in the inverted position using two control actions: the gyroscopic torque and the torque produced by the reaction wheel. Thus, Figure 2 presents the results for taking the VSCMG pendulum from the downward position ($\theta = 180°$) to the upward position ($\theta = 0°$). Figure 2a shows the controlled angular position for the VSCMG pendulum. The gimbal rate is presented in Fig. 2b and it can be noticed that the gyroscopic torque is more used at the beginning of the simulation to start moving the pendulum and after some seconds the torque is mainly produced by the reaction wheel. Some differences between the desired rates (- - -) at the beginning of the simulations can be noticed because for the CMG torque u_g, a saturation limit of 2 N.m was introduced what makes the control more realistic since it is intended to build an experimental device to evaluate the behavior in a practical experiment. The reaction wheel angular velocity is shown in Fig. 2c where it can be noticed that after the pendulum reaches the equilibrium in the inverted position the wheel speed is constant. So far, the results have shown that it is possible to control a pendulum using a VSCMG driving it from the downward position to the inverted position using two control actions.

FINAL REMARKS

This paper has presented a new configuration of a pendulum that has a VSCMG mechanism coupled to it. A nonlinear controller was designed using the Lyapunov control theory to perform the attitude control of this new pendulum configuration. Thus, it could be noticed that the use of two control actions, gyroscopic torque and the torque provided by the reaction wheel, was very effective to take the pendulum from the downward position to its upward position controlling it in the unstable equilibrium point.



(a) Pendulum angular position $\theta(t)$

(b) Gimbal rate $\dot{\gamma}(t)$

(c) RW angular velocity $\dot{\psi}(t)$

Figure 2 – Simulation results for controlling the VSCMG in the inverted position ($\theta = 0°$). For sub-figures b and c line - - - indicates the desired rates.

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