AIAA 2008-6259

Locally Power-Optimal Spacecraft Attitude Control for Redundant Reaction Wheel Cluster

Hanspeter Schaub

AIAA/AAS Astrodynamics Specialist Conference
Honolulu, Hawaii, August 18–21, 2008

Locally Power-Optimal Spacecraft Attitude Control for Redundant Reaction Wheel Cluster

Hanspeter Schaub*

The attitude control problem of a rigid spacecraft containing a redundant set of reaction wheels is investigated. The classical solution path determines a control solution which requires the smallest set of motor torques. With small micro spacecraft the available power or the energy consumption are critical factors. A locally optimal attitude control strategy is developed which minimizes the instantaneous electrical power requirements. Degenerate conditions with several fly wheel speeds being zero are also investigated. The new control is able to reduce the amount of power and energy required by about 10%, while only marginally increasing the average required torque.

I. Introduction

To change and control the orientation or attitude of a spacecraft the actuation methods typically fall into the categories of fuel consuming thrusters,[1,2,3] internal momentum exchange devices requiring electrical power,[4,5] or external environmental torques due to the gravity gradient, atmospheric, or magnetic torques.[6,7,8] The attitude control of spacecraft continues to be a rich area of research with many new issues being investigated. While some papers focus on developing robust adaptive attitude control strategies using thrusters,[9] this paper focuses on the spacecraft attitude control using momentum exchange devices. These include the momentum or Reaction Wheels (RWs), the Control-Momentum Gyroscopes (CMGs), or the more recent Variable-Speed Control Moment Gyroscopes (VSCMGs). The RWs exert a torque onto the craft by spinning up or down the fly wheel.[10] These mechanically simple devices are often limited in the amount of torque they can produce, and have limits to which the fly wheel can be spun up to. CMG devices are essentially gimbaled RWs whose spin rate is held constant through the use of a local spin motor. The attitude control is produce by rotating or gimbaling the RW spin axis. The larger the spin rate is the larger the resulting gyroscopic torque will be. While mechanically more complex than RW clusters, CMG clusters can typically produce larger control torques. They are often used as the attitude control device for larger spacecraft such as the space-station, or when very rapid reorientations are required. The control laws of single-axis CMGs are also more complex and have geometric singularities which must be accounted for.[11,12,13] More recently the concept of a VSCMG has been proposed.[14,15] Here the momentum device can produce a control torque by both changing the RW spin speed, as well as gimbaling the RW spin axis. A properly configured VSCMG cluster will never encounter a geometric singularity where the required control torque cannot be produced. However, this is achieved at the increased power cost of having to use the RW to muscle through the singularity. Reference 16 discusses a method where the VSCMG null motion is exploited to keep the VSCMG cluster away from a traditional CMG singularity, and thus the VSCMG devices can operate in the more efficient CMG mode. The RW mode is only minimally used to nudge the gimbal angles into a desired geometry. A cluster of 4 VSCMG devices thus forms a highly redundant set of attitude control devices whose control could also be set to track alternate goals. For example, Reference 17 discusses how the VSCMG control could be used to track a required electrical power profile by slowly de-spinning the VSCMGs and harvesting the resulting electrical energy while the craft cannot operate the solar panels. This concept leads to a joint attitude and energy storage device.

The attitude control of small satellites contains its own set of challenges. The small craft often are very limited in the amount of on-board propellant, and thus cannot afford to use this fuel to perform the attitude control.[18,19] Instead

*Associate Professor, H. Joseph Smead Fellow, Aerospace Engineering Sciences Department, Colorado Center for Astrodynamics Research, University of Colorado at Boulder.

the use of momentum wheels is considered as a more energy efficient attitude control method.[20,21] However, note that the RW and CMG devices of a small satellite typically will operate at a much higher spin rate than those of a more typically sized spacecraft. Further, the amount of electrical power that a small satellite can produce is very limited. Due to the small size, there is little surface to vent the excess heat. The focus of this paper is the attitude control of a spacecraft which is limited in its available power and energy. In particular, it is assumed that the craft has a redundant set (more than 3) of RWs to control its attitude. Such redundant RW setups are common in that they provide additional robustness to individual RW failures. When determining the RW motor control torques, there are now an infinity of solutions available. The standard solution is to chose a simple minimum norm inverse and obtain the smallest RW motor torque vector. This solution is very practical if the RW torque capability are a strong performance limiting factor. However, considering the application of small spacecraft, this paper investigates alternate RW motor torque solutions where the local electrical power consumption is minimized instead of the motor torques. Not that the power consumption is not being optimized across a given maneuver. Instead, and an instantaneous power-optimal feedback control solution is investigated.

The design and control of optimal RW clusters has been discussed in previous publications, but not yielding a locally power-optimal feedback control law. For example, Reference 22 discusses the optimal RW alignment to produce optimal RW torque or power solutions. Vadali in Reference 23 discusses optimal control solutions which minimize various performance aspects across a maneuver. Being an optimal control solution, such control torque calculations required knowledge of both the initial and final attitude states. In contrast, the feedback control discussed in this paper only requires the instantaneous attitude states. It will not lead to maneuver optimal solutions, but does provide simpler to implement feedback control strategies. The power optimal RW spacecraft attitude control is discussed by Skaar and Kraige in References 24 and 25. However, here too the end solutions are optimal control strategies which require initial state information to be solved apriori to the maneuver being performed.

The paper is setup as follows. First the equations of motion of a rigid spacecraft containing N reaction wheels is developed and the notation used is explained. Next, the standard minimum motor torque attitude feedback control solution is developed and discussed. Finally the analytical closed-form solution of the locally power-optimal redundant RW control is developed. Degenerate conditions where some of the RW have zero spin rates are investigated. Numerical simulations illustrate and compare the new locally power-optimal feedback control to the traditional torque-optimal solution. Of interest is how much the instantaneous power and total energy expenditure is reduced by using this alternate RW motor torque strategy.

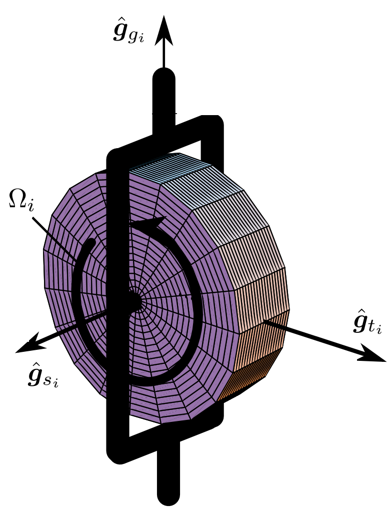


Figure 1: Illustration of a Reaction Wheel Coordinate Frame

Problem Statement

The spacecraft is assumed to be composed of a rigid body $\mathcal{B}$ containing N variable-speed reaction wheels. The spacecraft body fixed coordinate frame is given by $\mathcal{B} : \{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$. The orientation of each Reaction Wheel (RW) is defined through the body fixed wheel frames $\mathcal{G}_i : \{\hat{g}_{s_i}, \hat{g}_{t_i}, \hat{g}_{g_i}\}$ illustrated in Figure 1. If this attitude control device were a single-gimbal control moment gyroscope (CMG) device, then the spinning disk would be allowed to rotate about the $\hat{g}_{g_i}$ axis. For the RW case the disk is spinning with a speed Ω_i about the spin axis $\hat{g}_{s_i}$.

Reference 10 develops the attitude equations of motion for such a system. The differential equations of motion are given by

$$[I]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}][I]\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}][G_s]\boldsymbol{h}_s - [G_s]\boldsymbol{u}_s + \boldsymbol{L} \tag{1}$$

where $\boldsymbol{L}$ is an external torque vector, and $[\tilde{\boldsymbol{\omega}}]$ is defined as matrix equivalent of a vector cross product using

$$[\tilde{\boldsymbol{\omega}}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

To express the body angular velocity vector in spacecraft body frame $\mathcal{B}$ or wheel frame $\mathcal{G}_\rangle$ vector components, the following notation is used:

$$\boldsymbol{\omega} = \omega_{s_i}\hat{\boldsymbol{g}}_{s_i} + \omega_{t_i}\hat{\boldsymbol{g}}_{t_i} + \omega_{g_i}\hat{\boldsymbol{g}}_{g_i} = \omega_1\hat{\boldsymbol{b}}_1 + \omega_2\hat{\boldsymbol{b}}_2 + \omega_3\hat{\boldsymbol{b}}_3 \tag{3}$$

The 3×3 matrix $[I]$ is the constant inertia system matrix defined as

$$[I] = [I_s] + \sum_{i=1}^{N} \left(J_{t_i}\hat{\boldsymbol{g}}_{t_i}\hat{\boldsymbol{g}}_{t_i}^T + J_{t_i}\hat{\boldsymbol{g}}_{g_i}\hat{\boldsymbol{g}}_{g_i}^T \right) \tag{4}$$

where $[I_s]$ is the inertia matrix of the rigid spacecraft itself. Due to symmetry the wheel principal inertias are given by $(J_{s_i}, J_{t_i}, J_{t_i})$. The wheel frames $\mathcal{G}_i$ are assumed be principal coordinate frames for the RW disks such that the wheel inertias are defined through

$$[I_{W_i}] = J_{s_i}\hat{\boldsymbol{g}}_s\hat{\boldsymbol{g}}_s^T + J_{t_i}\hat{\boldsymbol{g}}_t\hat{\boldsymbol{g}}_t^T + J_{t_i}\hat{\boldsymbol{g}}_g\hat{\boldsymbol{g}}_g^T \tag{5}$$

Please note that this $[I]$ inertia matrix definition includes the inertia of the spacecraft, the $\hat{\boldsymbol{g}}_{t_i}$ and $\hat{\boldsymbol{g}}_{g_i}$ components of the wheel inertia, as well as the inertia terms due to the RW center of masses being offset from the spacecraft body center of mass. The RW inertias J_{s_i} about the spin axis are factored out of this inertia matrix expression.

The N-dimensional torque vector $\boldsymbol{u}_s$ is the RW torque control vector and is defined as

$$\boldsymbol{u}_s = \begin{pmatrix} \vdots \\ u_{s_i} \\ \vdots \end{pmatrix} \tag{6}$$

where u_{s_i} are the i$^{\text{th}}$ RW motor torques defined through

$$u_{s_i} = J_{s_i}\left(\dot{\Omega}_i + \hat{\boldsymbol{g}}_{s_i}^T\dot{\boldsymbol{\omega}}\right) \tag{7}$$

The N-dimensional momentum vector $\boldsymbol{h}_s$ is defined as

$$\boldsymbol{h}_s = \begin{pmatrix} \vdots \\ J_{s_i}\left(\omega_{s_i} + \Omega_i\right) \\ \vdots \end{pmatrix} \tag{8}$$

Finally, the $3 \times N$ projection matrix $[G_s]$ is given by

$$[G_s] = [\hat{\boldsymbol{g}}_{s_1} \cdots \hat{\boldsymbol{g}}_{s_N}] \tag{9}$$

Note that to numerically evaluate Eq. (1) it is assumed that all vector and matrix components have been taken with respect to the same coordinate frame before performing matrix algebra.

The rotational kinetic energy T of a rigid spacecraft with N RWs is given by[10]

$$T = \frac{1}{2}\boldsymbol{\omega}^T[I_s]\boldsymbol{\omega} + \frac{1}{2}\sum_{i=1}^{N} J_{s_i}\left(\Omega_i + \omega_{s_i}\right)^2 + J_{t_i}\omega_{t_i}^2 + J_{t_i}\omega_{g_i}^2 \tag{10}$$

The kinetic energy rate, also known as the work rate or power equation, is found after differentiating Eq. (10), or simply by applying the Work-Energy-Rate principle,[26] to be

$$\dot{T} = \boldsymbol{\omega}^T \boldsymbol{L} + \sum_{i=1}^{N} \Omega_i u_{s_i} \tag{11}$$

Therefore, in the absence of an external torque vector $\boldsymbol{L}$, the power P_i required by each RW motor is given by

$$P_i = \Omega_i u_{s_i} \tag{12}$$

II. Minimum Torque Redundant Reaction Wheel Control Law

To control the spacecraft attitude, a feedback control law $\boldsymbol{u}_s$ is required to stabilize the craft to a desired orientation. The following development will not depend on the specific type of attitude feedback control law that is chosen. The difference arise in how this desired control torque is mapped into the commanded RW motor torques u_{s_i}.

To setup the redundant RW control problem, let $\boldsymbol{\sigma}$ be a set of Modified Rodrigues Parameters (MRPs)[27,28,29,30,10] which define the orientation of the body frame $\mathcal{B}$ with respect to a reference frame $\mathcal{R}$. The vector $\boldsymbol{\omega}$ is the body angular velocity of the spacecraft body, while ω_r is the desired reference angular velocity vector. The angular velocity error vector $\delta\boldsymbol{\omega}$ be defined as

$$\delta\boldsymbol{\omega} = \boldsymbol{\omega} - \boldsymbol{\omega}_r \tag{13}$$

To develop a stabilizing feedback control law for this attitude trajectory tracking problem, the following positive definite Lyapunov function V can be used:[10,29,1]

$$V(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I]\delta\boldsymbol{\omega} + 2K \ln\left(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma}\right) \tag{14}$$

After setting the time derivative of V equal to the negative semi-definite function

$$\dot{V} = -\delta\boldsymbol{\omega}[P]\delta\boldsymbol{\omega} \tag{15}$$

and substituting the equations of motion in Eq. (1), the required RW motor torque vector is defined through the constraint:

$$[G_s]\boldsymbol{u}_s = K\boldsymbol{\sigma} + [P]\delta\boldsymbol{\omega} - [\tilde{\boldsymbol{\omega}}]\left([I]\boldsymbol{\omega} + [G_s]\boldsymbol{h}_s - \boldsymbol{\omega}_r\right) - [I]\left(\dot{\boldsymbol{\omega}}_r - \boldsymbol{\omega} \times \boldsymbol{\omega}_r\right) + \boldsymbol{L} = \boldsymbol{L}_r \tag{16}$$

The left hand side of Eq. (16) contains a projection matrix $[G_s]$ which maps the actual RW motor torques in the actual torque exerted onto the vehicle. The right hand side of Eq. (16) is the reference control torque $\boldsymbol{L}_r$ that is required by the chosen feedback control strategy.

$$[G_s]\boldsymbol{u}_s = \boldsymbol{L}_r \tag{17}$$

Note that while there are an infinity of $\boldsymbol{u}_s$ choices which will produce the required torque, all control solutions will yield the same attitude closed loop dynamics with the same $\boldsymbol{\sigma}$ and $\delta\boldsymbol{\omega}$ time histories. However, the RW spin rates Ω_i will be different for different choices the RW torques.

If the matrix $[G_s]$ is full rank then the RW cluster can produce the required control torque $\boldsymbol{L}_r$ exactly. If this projection matrix is not full rank, then $\boldsymbol{L}_r$ can only be partially produced. This latter situation is a common singular configuration with single-gimbal CMG devices where at particular gimbal angles no set of gimbal angle rates will produce the required control torque. With RW clusters the geometry of the spin axis is general chosen such that the $\hat{\boldsymbol{g}}_{s_i}$ vectors span the three-dimensional space, and thus $[G_s]$ is full rank. Further, for the RW cluster control problem $[G_s]$ is a constant matrix. If more than 3 RWs are employed, then the $[G_s]$ matrix will contain a non-empty nullspace, resulting in an infinity of u_{s_i} combination which produce the required control torque $\boldsymbol{L}_r$.

Please note that *all* RW cluster control formulations can be written in the compact form shown in Eq. (17). If a different control strategy compared to the solution based on the Lyapunov function in Eq. (14) is chosen, then only the required torque $\boldsymbol{L}_r$ definition will change. For redundant RW setups, the typical RW motor torque strategy employed seeks for the minimum norm solution $\boldsymbol{u}_s^*$ which requires the smallest absolute motor torques. This solution is given by

$$\boldsymbol{u}_s^* = [G_s]^T \left([G_s][G_s]^T\right)^{-1} \boldsymbol{L}_r \tag{18}$$

This solution is convenient when the RW motor torque limits are of concern. This is often the case when the RW cluster is controlling the attitude of a large and massive spacecraft. While RW devices are not encumber with singular orientations such as are found with CMG clusters, they are limited by maximum spin speed Ω_i and the torque they can produce.

Of interest is exploring an alternate method of mapping the required control torque $\boldsymbol{L}_r$ into the RW motor torque vector $\boldsymbol{u}_s$. Instead of minimizing the instantaneous torque requirement, the RW motor power requirements will be investigated instead.

III. Power-Optimal Control Formulation

Small satellite concepts are typically very limited in the amount of electrical power that they can produce, or the amount of energy that they can store. For example, see the SNAP-I nanosatellite discussed in Reference 31. Such craft concepts are limited in how much electrical power they can provide while radiating out excess thermal energy. Reference 20 discusses experimental results of a cluster of miniature CMG devices to control the small spacecraft orientation. A key concern here is the peak power requirements, and the total energy consumed for a maneuver.

The typical RW cluster control law solution in Eq. (18) which minimizes the instantaneous torque may not be the ideal solution for a small satellite with strong power and energy consumption limitations. This section investigates an alternate method of mapping the required control torque $\boldsymbol{L}_r$ to the RW control torques $\boldsymbol{u}_s$ in Eq. (17). Note that either control strategy uses the same Lyapunov function in Eq. (14) and have the same required torque $\boldsymbol{L}_r$ expression, they only differ in resulting motor torque computation. Let R be the rank of the $3 \times N$ projection matrix $[G_s]$, while $M = N - R$ is the degree of redundancy in the RW cluster. The minimum RW motor torque solution $\boldsymbol{u}^*$ is only one of an infinity of solutions. Let the general motor torque vector be expressed as

$$\boldsymbol{u}_s = \boldsymbol{u}^* + [N]\boldsymbol{t} \tag{19}$$

where $[N]$ is the $N \times M$ the null-space matrix of $[G_s]$ satisfying

$$[G_s][N] = [0_{3\times M}] \tag{20}$$

The vector $\boldsymbol{t}$ contains the M null-space scaling parameters through

$$\boldsymbol{t} = \begin{pmatrix} t_1 & \cdots & t_M \end{pmatrix}^T \tag{21}$$

For a given RW cluster the goal is to find the null-space scaling parameters t_i such that the instantaneous power consumption is minimized. The total instantaneous mechanical power P required is given by

$$P = \sum_{i=1}^{N} \Omega_i u_{s_i} = \sum_{i=1}^{N} P_i \tag{22}$$

However, note that the P_i components can be positive or negative. A positive power P_i means that the i^{th} RW device requires a power input to achieve the maneuver. A negative power implies that the RW could return mechanical energy to the cluster. For example, consider the case where the spin wheel must be decelerated. Instead of applying brakes which would convert the mechanical spin energy into heat, it could be possible to use a dynamo device which could decelerate the wheel and convert its mechanical energy into stored electrical energy. This energy could then be used to accelerate other wheels. In this case it would make sense to try to minimize the total instantaneous mechanical power usage in Eq. (22). However, such energy capture mechanisms are not typically employed with RW devices. Instead a different cost function must be used to account for both acceleration and deceleration contributing to the total electrical power requirement.

Let $\boldsymbol{P} = \begin{pmatrix} P_1 & \cdots & P_N \end{pmatrix}^T$ be a vector containing the RW powers P_i. Using Eq. (12), the power vector can also be expressed as

$$\boldsymbol{P} = [\Omega]\boldsymbol{u}_s \tag{23}$$

where the diagonal matrix $[\Omega]$ is defined as

$$[\Omega] = \text{diag}(\Omega_i) \tag{24}$$

Let the positive cost function J be defined in terms of the L_2 norm of $\boldsymbol{P}$:

$$J = \frac{1}{2}\left(|\boldsymbol{P}|_2\right)^2 = \frac{1}{2}\sum_{i=1}^{N} P_i^2 = \frac{1}{2}\boldsymbol{P}^T\boldsymbol{P} \tag{25}$$

This cost functions takes into account that both acceleration and deceleration of RWs requires electrical power. Next the torque vector $\boldsymbol{u}_s$ must be found which will minimize this cost function. Using Eq. (19) and (23) the cost function J is rewritten as

$$J = \frac{1}{2}\Big([\Omega](\boldsymbol{u}_s^* + [N]\boldsymbol{t})\Big)^T\Big([\Omega](\boldsymbol{u}_s^* + [N]\boldsymbol{t})\Big) \tag{26}$$

A necessary condition for a minimum of J with respect to the null-space scaling parameter is

$$\frac{\partial J}{\partial \boldsymbol{t}} = \Big([\Omega](\boldsymbol{u}_s^* + [N]\boldsymbol{t})\Big)^T[\Omega][N] = \boldsymbol{0} \tag{27}$$

Carrying out the matrix algebra leads to

$$\underbrace{[N]^T[\Omega]^2[N]}_{[A]}\boldsymbol{t} = -[N]^T[\Omega]^2\boldsymbol{u}_s^* \tag{28}$$

Before solving for $\boldsymbol{t}$ the invertibility of $[A]$ must be investigated. The null-space matrix $[N]$ is expressed using the M-dimensional vectors $\boldsymbol{n}_i$ as

$$[N] = \begin{bmatrix} \boldsymbol{n}_1^T \\ \vdots \\ \boldsymbol{n}_N^T \end{bmatrix} \tag{29}$$

Note that none of the $\boldsymbol{n}_i$ vectors are a zero vector. The $M\times M$ matrix $[A]$ is then written as

$$[A] = \sum_{i=1}^{N}\Omega_i^2\boldsymbol{n}_i\boldsymbol{n}_i^T \tag{30}$$

Because $[N]$ has rank M through its definition as the null-space matrix of $[G_s]$, the rank of $[A]$ is also M if the RW spin rates are non-zero with $\Omega_i^2 > 0$. In fact, the matrix $[A]$ has rank M and is invertible if at least M RWs have a non-zero spin rate. For example, if there are 4 RWs on the spacecraft, then the null-space $[N]$ of $[G_s]$ is a 4×1 matrix with M = 1. Because $[N]$ cannot contain columns or rows of zeros, all components of $[N]$ are non-zero in this case. Here $[A]$ is invertible as long as at least one RW has a non-zero speed. If the craft has 5 RWs, then then at least 2 RWs will have to have non-zero spin rates. If $[A]$ is invertible, then the optimal null-space scaling parameter vector $\hat{\boldsymbol{t}}$ is given by

$$\hat{\boldsymbol{t}} = -([N]^T[\Omega]^2[N])^{-1}[N]^T[\Omega]^2\boldsymbol{u}_s^* \tag{31}$$

Setting $\partial J/\partial\boldsymbol{t} = \boldsymbol{0}$ is only a necessary condition for the power-optimal solution. To guarantee a minimum power solution $\partial J^2/\partial\boldsymbol{t}^2 > 0$ must be a positive definite matrix. Differentiating Eq. (27) with respect to $\boldsymbol{t}$ yields

$$\frac{\partial J^2}{\partial \boldsymbol{t}^2} = [N]^T[\Omega]^2[N] \tag{32}$$

Using the $[N]$ definition in Eq. (29) this is rewritten as

$$\frac{\partial J^2}{\partial \boldsymbol{t}^2} = \sum_{i=1}^{N}\Omega_i^2\boldsymbol{n}_i\boldsymbol{n}_i^T \tag{33}$$

which yields a positive definite matrix by inspection for the general case with $\Omega_i \neq 0$. Thus the solution in Eq. (31) provides the null-space scaling parameters yielding a minimum instantaneous power control.

What occurs if the $[A]$ matrix is not invertible? First, consider the simple case where all the RWs are at rest with $\Omega_i = 0$. Studying Eq. (26) it is apparent that the power cost function is zero regardless of which torque solution is used. Any torque solution in Eq. (19) would provide a power optimal solution. In this case it would make sense to simply use the minimum torque solution $\boldsymbol{u}_s^*$.

Next the scenario is investigated where some Ω_i are non-zero, yet the $[A]$ matrix is not full rank. Let R be the number of non-zero RW spin rates Ω_i, where $R < M$ to guarantee that $[A]$ is not invertible. Without loss of generality let us assume that only the first R craft have non-zero Ω_i. Equation (28) is satisfied if a vector $\boldsymbol{t}$ is chosen such that

$$\boldsymbol{n}_i^T \boldsymbol{t} = -u_{s_i}^* \qquad \text{for } i = 1, \cdots, R \tag{34}$$

Using Eq. (30) the power-optimal scaling parameter condition in Eq. (28) is rewritten as

$$[A]\boldsymbol{t} = \left(\Omega_1^2 \boldsymbol{n}_1 \boldsymbol{n}_1^T + \cdots + \Omega_R^2 \boldsymbol{n}_R \boldsymbol{n}_R^T\right)\boldsymbol{t} = -\Omega_1^2 \boldsymbol{n}_1 u_{s_1}^* - \cdots - \Omega_R^2 \boldsymbol{n}_R u_{s_R}^* \tag{35}$$

where $u_{s_i}^*$ is the i^{th} components of $\boldsymbol{u}_s^*$. Because the square matrix $[A]$ is not full-rank in this scenario, it is not possible to solve this equation for a unique $\boldsymbol{t}$. Instead, there are an infinity of scaling parameters which yield the desired power optimal solution. A simple solution to Eq. (35) is to determine the minimum norm solution to $\boldsymbol{t}$. Let the $R \times M$ matrix $[\mathcal{N}]$ be defined as

$$[\mathcal{N}] = \begin{bmatrix} \boldsymbol{n}_1^T \\ \vdots \\ \boldsymbol{n}_R^T \end{bmatrix} \tag{36}$$

and $\boldsymbol{\mathcal{U}}_s = \begin{pmatrix} u_{s_1}^* & \cdots & u_{s_R}^* \end{pmatrix}^T$, then the desired null-space scaling parameter vector $\boldsymbol{t}$ is determined using

$$\hat{\boldsymbol{t}} = -\mathcal{N}^T([\mathcal{N}][\mathcal{N}]^T)^{-1}\boldsymbol{\mathcal{U}}_s \tag{37}$$

for this degenerate scenario with an infinity of solutions.

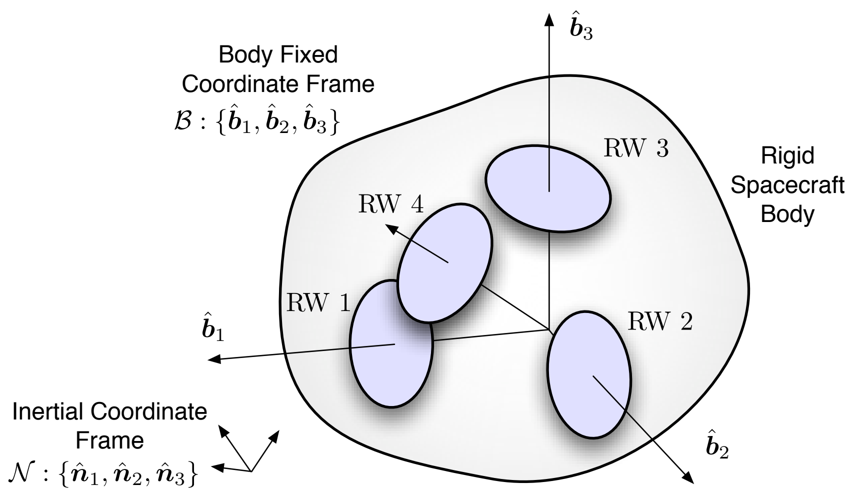
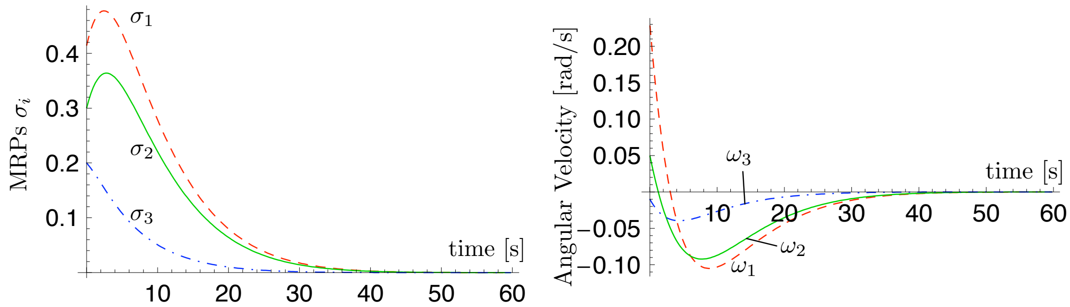


Figure 2: Spacecraft Illustration containing 4 Reaction Wheels

IV. Numerical Simulations

Numerical simulations of a spacecraft containing 4 RWs are performed to compare the minimum-torque attitude control solution in Eq. (18) to the minimum-power control proposed in Eq. (31). As illustrated in Figure 2, the 4 RW spin axis $\hat{\boldsymbol{g}}_{s_i}$ as setup as follows in spacecraft body frame $\mathcal{B}$ coordinates:

$$\hat{\boldsymbol{g}}_{s_1} = {}^{\mathcal{B}}\begin{bmatrix}1\\0\\0\end{bmatrix} \qquad \hat{\boldsymbol{g}}_{s_2} = {}^{\mathcal{B}}\begin{bmatrix}0\\1\\0\end{bmatrix} \qquad \hat{\boldsymbol{g}}_{s_3} = {}^{\mathcal{B}}\begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \hat{\boldsymbol{g}}_{s_4} = \frac{1}{\sqrt{3}}{}^{\mathcal{B}}\begin{bmatrix}1\\1\\1\end{bmatrix} \tag{38}$$



a) Orientation

b) Body Angular Velocity

Figure 3: Spacecraft Attitude Tracking Errors.

This standard redundant RW configuration has the first three RW spin axes aligned with the principal spacecraft body axes, while a 4$^{\text{th}}$ wheel is aligned diagonally to the others. In this setup the loss of any RW can be compensated for by the remaining three RWs. The 1×4 null-space matrix $[N]$ of $[G_s]$ is expressed as

$$[N] = \begin{bmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & 1 \end{bmatrix} \tag{39}$$

The spacecraft inertia matrix is given by

$$[I] = \text{diag}(86.215, 85.070, 113.565)\ \text{kg}\,\text{m}^2 \tag{40}$$

while the RW spin axis inertia is $J_s = 0.13$ kg m^2.

The reference attitude is set to be that of the inertial frame $\mathcal{N}$. The simulations demonstrate the response of a regular problem. The initial spacecraft state vectors are

$$\boldsymbol{\sigma}(t_0) = (0.414, 0.3, 0.2) \tag{41}$$

$$\boldsymbol{\omega}(t_0) = (0.23, 0.05, -0.01)\ \text{rad/s} \tag{42}$$

The reaction wheel rates Ω_i are all set to 40 rpm initially. The minimum-power attitude control law has a more noticeable performance difference to the minimum-torque solution if the reactions have a non-zero spin rate. While the nominal RW operating condition is to have the wheels speeds near zero, the Ω's do not remain near zero as they compensate for persistent external torques. The control feedback gains are set to

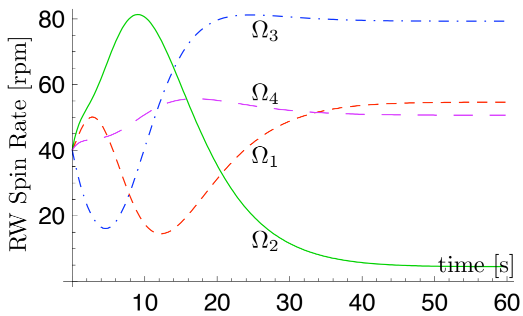
$$K = 0.1\ \text{Nm} \qquad P = 0.3\ \text{Nms}$$

The numerical simulations are run for 60 seconds each. The attitude response is illustrated in Figure 3. Regardless of the choice of $\boldsymbol{u}_s$ which produces the required torque $\boldsymbol{L}_r$ in Eq. (17), the attitude closed loop equations are the same. The feedback gains are chosen such that the attitude errors decay in a near critically damped manner.

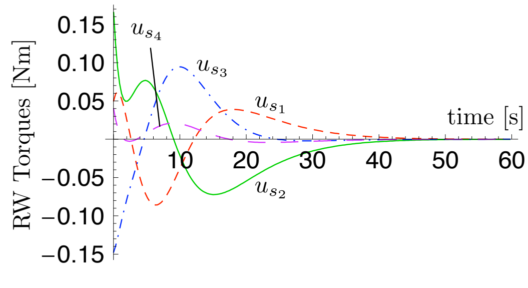
Where the minimum-torque (Case 1) and minimum-power (Case 2) RW control solutions differ is in internal RW speeds, the applied motor torques, and the instantaneous power requirement. Of interest is how much instantaneous power requirement is changed in Case 2, and how much energy can be saved. The Case 1 RW spin rates and motor torques are illustrated in Figures 4(a) and 4(b). To return the spacecraft attitude to that of the inertial frame, the control requires the initial 40 rpm Ω_i rates to change substantially. The equivalent states for Case 2 are illustrated in Figures 4(c) and 4(d). The overall response is similar to that of Case 1. However, the rate of Ω_2 actually changes its sign in this scenario. The individual torque trajectories vary slightly from those of Case 1, but not substantially.

The total electrical power required throughout the regulation maneuver is illustrated in Figure 4(e). Note that the Case 2 requires lower power levels at all times. This is not guaranteed to occur for all initial conditions. The minimum-power attitude control law only minimizes the required power at a particular time, and not for an entire maneuver. However, the behavior shown was typically seen for all the test runs attempted.

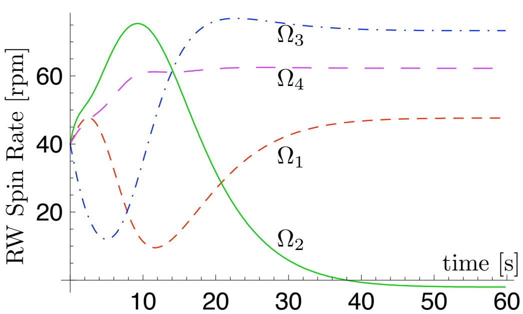
The attitude control of small spacecraft is often limited by the limited peak electrical power available to the control system. Note that both cases have about the same peak power requirement at the beginning of the simulation. The minimum-power solution did not substantially reduce this. However, the new control did reduce the average power requirement from about 0.203 J/s to 0.182 J/s, a 10.5% reduction. As a result case 2 uses 10.5% less energy to perform



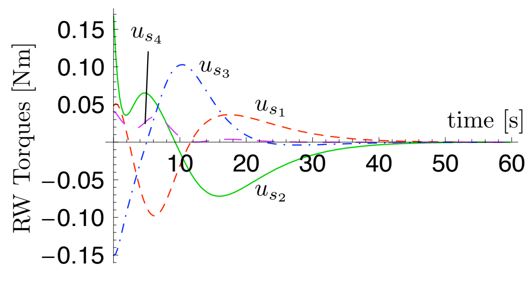
a) Reaction Wheel Spin Rates for Case 1



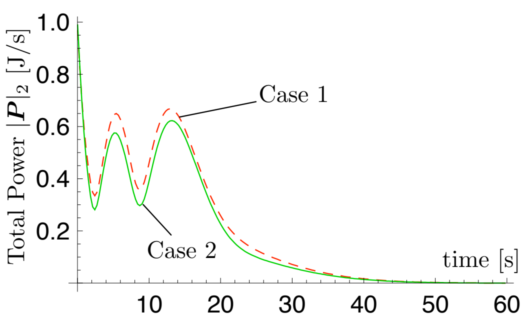
b) Reaction Wheel Motor Torques for Case 1



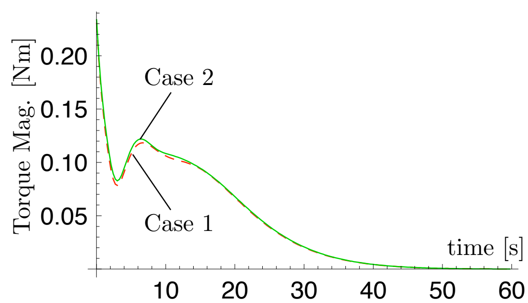
c) Reaction Wheel Spin Rates for Case 2



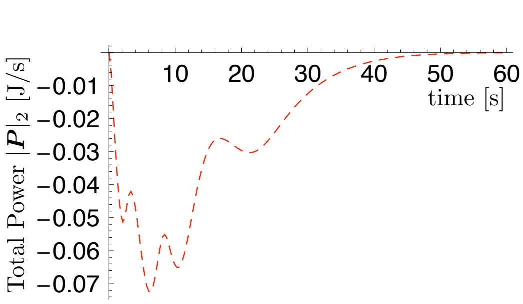
d) Reaction Wheel Motor Torques for Case 2



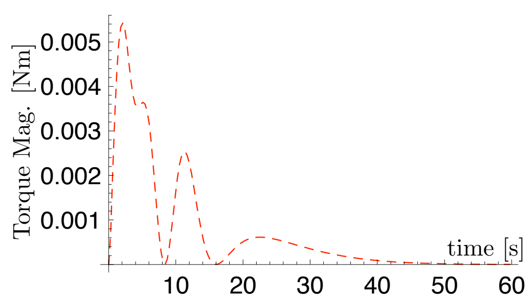
e) Power Requirements



f) Torque Vector Magnitude



g) Difference in Power Requirements



h) Difference in Torque Vector Magnitude

Figure 4: Comparison of Minimum-Torque Control (Case 1) and Minimum-Power Control (Case 2) Performances.

the same attitude regulation maneuver. Energy storage is also limited with the small satellites, and such an energy savings is significant.

The cost to achieve these power and energy saving is a slightly increased torque. Figure 4(f) shows the magnitude of the RW torque vector $\boldsymbol{u}_s$ during the maneuver. As expected, the total torques required for case 2 are larger than those of case 1. However, they are only larger by a very small margin. This behavior is common across all the initial conditions tested. The differences in the total RW power and torque requirements for this maneuver are illustrated in Figures 4(g) and 4(h). The maximum percent increase in torque is only about 6.27%, while the reduction in total energy required is over 10%.

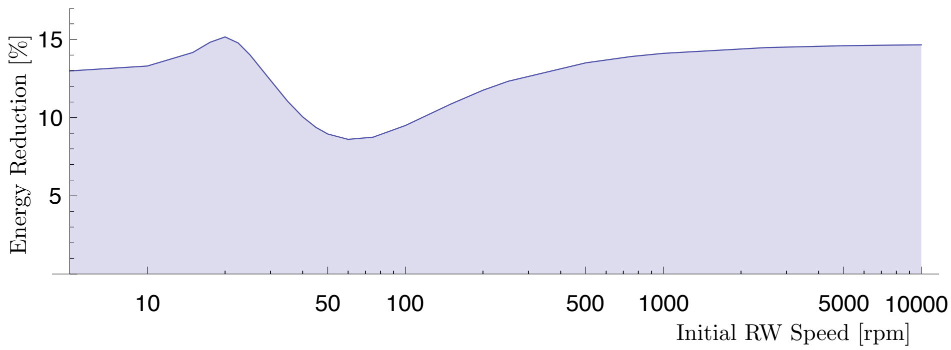


Figure 5: Maneuver-Wide Energy Usage Reduction for Various Initial RW Speeds.

The energy savings for various initial reaction wheel spin speeds are shown in Figure 5. For small initial fly wheel speeds the resulting attitude control maneuver will strongly influence how much energy is saved. However, as the initial RW speeds increased to larger values, then the energy saving settle down between 14–15%.

V. Conclusion

The classical minimum torque attitude control law for a redundant cluster of reaction wheels is revisited to examine locally power-optimum solutions. The reaction wheel redundancy creates a null-space in the fly wheel motor torque solution. An analytical solution is provided which determines which solution in the null-space will provide the smallest electrical power requirement at the current time step. This control does not provide for global maneuver-wide optimal power solutions. However, the new control strategy can provide a 10% or better energy saving for a minimal increase in the average torque used. Degenerate cases where several fly wheels have a zero spin speed are also investigate. Here the local minimum power solution is no longer unique and an alternate closed form analytical expression is provided to determine a set of reaction wheel motor torques. Future research will investigate the more complicated scenario of the spacecraft containing a cluster of CMG or variable speed CMG devices.

References

[1] Schaub, H., Robinett, R. D., and Junkins, J. L., "Globally Stable Feedback Laws for Near-Minimum-Fuel and Near-Minimum-Time Pointing Maneuvers for a Landmark-Tracking Spacecraft," *Journal of the Astronautical Sciences*, Vol. 44, No. 4, 1996, pp. 443–466.

[2] Tsiotras, P. and Longuski, J. M., "Spin-Axis Stabilization of Symmetric Spacecraft with Two Control Torques," *Systems & Control Letters*, Vol. 23, 1994, pp. 395–402.

[3] Tsiotras, P., "New Control Laws for the Attitude Stabilization of Rigid Bodies," *13th IFAC Symposium on Automatic Control in Aerospace*, Palo Alto, CA, Sept. 12–16 1994, pp. 316–321.

[4] Dwyer, T. A. W. I., "Exact Nonlinear Control of Spacecraft Slewing Maneuvers with Internal Momentum Transfer," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, 1986, pp. 240–247.

[5] Lawrence, D. A. and Holden, T. E., "Essentially Globally Asymptotically Stable Nutation Control Using a Single Reaction Wheel," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 6, Nov.–Dec. 2007, pp. 1783–1793.

[6] Makovec, K. L., *A Nonlinear Magnetic Controller for Three-Axis Stability of Nanosatellites*, Master's thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA, July 2001.

[7] Silani, E. and Lovera, M., "Magnetic Spacecraft Attitude Control: A Survey and Some New Results," *Control Engineering Practices*, Vol. 13, No. 3, March 2002, pp. 357–371.

[8] Yan, H., Alfriend, K. T., and Ross, M. I., "Three-Axis Attiude Control Using Pseudopectral Control Law," *AAS/AIAA Astrodynamics Specialist Conference*, Lake Tahoe, CA, Aug. 7–11 2005, Paper No. 05–417.

[9] Wallsgrove, R. J. and Akella, M. R., "Globally Stabilizing Saturated Attitude Control in the Presence of Bounded Unknown Disturbances," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 5, 2005, pp. 957–963.

[10] Schaub, H. and Junkins, J. L., *Analytical Mechanics of Space Systems*, AIAA Education Series, Reston, VA, October 2003.

[11] Ford, K. A. and Hall, C. D., "Singular Direction Avoidance Steering for Control-Moment Gyros," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 648–656.

[12] Wie, B., "Singularity Escape/Avoidance Steering Logic for Control Moment Gyro Systems," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 5, 2005, pp. 948–956.

[13] Bedrossian, N. S., Paradiso, J., Bergmann, E. V., and Rowell, D., "Steering law design for redundant single-gimbal control moment gyroscopes," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 6, 1990, pp. 1083–1089.

[14] Ford, K. A. and Hall, C. D., "Flexible Spacecraft Reorientations Using Gimbaled Momentum Wheels," *Journal of the Astronautical Sciences*, Vol. 49, No. 3, 2001, pp. 421–441.

[15] Schaub, H., Vadali, S. R., and Junkins, J. L., "Feedback Control Law for Variable Speed Control Moment Gyroscopes," *Journal of the Astronautical Sciences*, Vol. 45, No. 3, July–Sept. 1998, pp. 307–328.

[16] Schaub, H. and Junkins, J. L., "Singularity Avoidance Using Null Motion and Variable-Speed Control Moment Gyros," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 1, Jan.–Feb. 2000, pp. 11–16.

[17] Yoon, H. and Tsiotras, P., "Singularity Analysis of Variable-Speed Control Moment Gyros," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 27, No. 3, 2004, pp. 374–386.

[18] Ma, K. B., Zhang, Y., Postrekhin, Y., and Chu, W.-K., "HTS bearings for space applications: reaction wheel with low power consuption for mini-satellites," *IEEE Transactions on Applied Superconductivity*, Vol. 13, No. 2, June 2003, pp. 2275–2278.

[19] Zhang, Y., Postrekhin, Y., Bui Ma, K., and Chu, W.-K., "Reaction wheel with HTS bearings for mini-satellite attitude control," *Superconductor Science Technology*, Vol. 15, May 2002, pp. 823–825.

[20] Lappas, V. J., Steyn, W. H., and Underwood, C., "Design and Testing of a Control Moment Gyroscope Cluster for Small Satellites," *AIAA Journal of Spacecraft and Rockets*, Vol. 42, No. 4, July–Aug. 2005, pp. 729–739.

[21] Richie, D. J., Lappas, V. J., and Palmer, P. L., "Sizing/Optimization of a Small Satellite Energy Storage and Attitude Control System," *AIAA Journal of Spacecraft and Rockets*, Vol. 44, No. 4, 2007, pp. 940–952.

[22] Bayard, D. S., "An Optimization Result With Application to Optimal Spacecraft Reaction Wheel Orientation Design," *Proceedings of the American Control Conference*, Arlington, VA, June 25–27 2001, pp. 1473–1478.

[23] Vadali, S. R. and Junkins, J. L., "Spacecraft large angle rotational maneuvers with optimal momentum transfer," *Journal of the Astronautical Sciences*, Vol. 31, 1983, pp. 217–235.

[24] Skaar, S. B. and Kraige, L. G., "Single-Axis Spacecraft Attitude Maneuvers Using an Optimal Reaction Wheel Power Criterion," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 5, No. 5, Sept.–Oct. 1982, pp. 543–544.

[25] Skaar, S. B. and Kraige, L. G., "Large-Angle Spacecraft Attitude Maneuvers Using an Optimal Reaction Wheel Power Criterion," *Journal of the Astronautical Sciences*, Vol. 32, No. 1, 1984, pp. 47–61.

[26] Oh, H. S., Vadali, S. R., and Junkins, J. L., "On the Use of the Work-Energy Rate Principle for Designing Feedback Control Laws," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, 1992, pp. 272–277.

[27] Marandi, S. R. and Modi, V. J., "A Preferred Coordinate System and the Associated Orientation Representation in Attitude Dynamics," *Acta Astronautica*, Vol. 15, No. 11, 1987, pp. 833–843.

[28] Wiener, T. F., *Theoretical Analysis of Gimballess Inertial Reference Equipment Using Delta-Modulated Instruments*, Ph.D. dissertation, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, March 1962.

[29] Tsiotras, P., "Stabilization and Optimality Results for the Attitude Control Problem," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, 1996, pp. 772–779.

[30] Schaub, H. and Junkins, J. L., "Stereographic Orientation Parameters for Attitude Dynamics: A Generalization of the Rodrigues Parameters," *Journal of the Astronautical Sciences*, Vol. 44, No. 1, 1996, pp. 1–19.

[31] Steyn, W., Hashida, Y., and Lappas, V., "An Attitude Control System and Commissioning Results of the SNAP-1 Nanosatellite," *14th AIAA/USU Conference on Small Satellites*, August 2000, Paper No. SSC00-VIII-8.