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# DELAYED FEEDBACK ATTITUDE CONTROL USING NEURAL NETWORKS AND LYAPUNOV-KRASOVSKII FUNCTIONALS

Ehsan Samiei,* Morad Nazari,* Eric A. Butcher,† and Hanspeter Schaub‡

This paper addresses the regulation control and stabilization problem of spacecraft attitude dynamics when there exists an unknown constant discrete delay in the measurements. Radial basis function neural networks are used to approximate the kinematics and inertial nonlinearities while a back propagation algorithm is employed to update neural network weights. By employing a Lyapunov-Krasovskii functional, a delay independent stability condition is obtained in terms of a linear matrix inequality, the solution of which gives the suitable controller gains. Finally, to show the effectiveness of the proposed controller, a set of simulations are performed and the results of the proposed control strategy are compared with results obtained using the method for delayed attitude control suggested by Ailon *et al.*[1]

## NOMENCLATURE

| | |
|---|---|
| $\vec{a}$ | additional robust term |
| $J$ | inertia matrix in principal coordinates |
| $\vec{u}$ | feedback control law |
| $V$ | Lyapunov candidate |
| $\vec{x}$, $\vec{\xi}$ | state space vectors |
| $\mathcal{M}$ | linear matrix inequality (LMI) |
| $W$, $\mathcal{V}$ | constant weight matrices |
| $\vec{\varepsilon}$ | neural network approximation error |
| $\eta$, $\kappa$ | constant controller gains |
| $\vec{\nu}$ | control variable |
| $\vec{\omega}$ | angular velocity vector |
| $\vec{\Phi}$ | a known vector-values function |
| $\vec{\sigma}$ | modified Rodriguez parameter set |
| $\tau$ | time delay (sec) |
| $(.)^+$ | pseudoinverse of $(.)$ |

*PhD student, Mechanical and Aerospace Engineering Department, New Mexico State University, Las Cruces, NM, 88003, USA.

†Associate Professor, Mechanical and Aerospace Engineering Department, AIAA member, New Mexico State University, Las Cruces, NM, 88003, USA.

‡Associate Professor, H. Joseph Smead Fellow, Aerospace Engineering Sciences Department, AAS Member, AIAA Associate Fellow, University of Colorado, Boulder, CO, 80309-0431, USA.

## INTRODUCTION

In recent decades, the problem of delayed feedback control has been subject to intensive research because of the wide range of applications in mechanical systems such as spacecraft attitude maneuvers, underwater vehicles, cooperative robot manipulation, etc. The attitude modeling problem depends on the choice of attitude parameters (coordinates) to represent the orientation of a rigid body relative to an inertial frame. When modeled as rigid bodies, spacecraft dynamics are studied based on certain kinematic and kinetic equations, where the kinematic equations depend on the selected attitude coordinates. While the kinetic equations are nonlinear, the kinematic equations can be either linear or nonlinear depending on the choice of attitude coordinates and hence different approaches have been developed for feedback control.

Attitude parameters can be either the rotation matrix, the principal angle and principal axis, Euler angles (EA), Euler parameters or quaternions (EPs), classical Rodriguez parameters (CRPs) or modified Rodriguez parameters (MRPs), among others[2] . The principal rotation vector is a basis for many attitude representations, but it has the disadvantage of the mathematical singularity for zero rotation. Therefore, this set is not suitable for regulation control where the reference state is the zero rotation. EAs are easy to visualize, but the reference frame is never more than 90 degrees rotation from a singularity. EPs, on the other hand, have considerable benefits including the facts that they are nonsingular and their corresponding kinematic equation is linear, and hence are widely used in spacecraft attitude studies. However, they are quite hard to visualize. CRPs, also known as Gibbs vector, reduce the EPs to a minimal three-parameter set. Based on their definition, they are much better suited for large spacecraft rotations than EAs. A further improved attitude representation, known as MRPs, moves the singularities to $360^{\mathrm{o}}$ rotation instead of $180^{\mathrm{o}}$ as in the case of CRPs.

Several control laws have been developed so far for the control of rigid body attitude. Tsiotras[3] designed an optimal controller, based on CRPs and MRPs, to minimize a quadratic cost function for a dynamical system, and investigated the stability of the system using Lyapunov functions. Sharma and Tewari[4] addressed the tracking control problem of a rigid asymmetric spacecraft by using Hamiltonian-Jacobi formulation, where MRPs were used as the coordinate set. The resulting controller was nonlinear and optimal and capable of tracking maneuvers with arbitrarily large initial conditions. Using geometric control, Lee and McClamroch[5] proposed a controller for both translational and rotational dynamics of a rigid body, where a Lie group variational integrator was implemented to treat the dynamics of the system. Using this strategy, the authors guaranteed that the geometry of the optimal solutions was preserved.

Few studies, however, to the authors' knowledge, have focused on delayed feedback control of spacecraft. Ailon *et al*[1] introduced a velocity free output-based controller for attitude regulation of a rigid spacecraft considering the effects of time delay in the system. The angular velocity feedback was replaced by the filtered attitude where the classical Rodriguez parameters (CRPs) were selected as the attitude parameters. Sufficient conditions for attitude stabilization of the spacecraft were studied . The time delay was assumed to be known in their study. Recently, new constructions of Lyapunov-Krasovskii (L-K) functionals have been developed for the stability analysis of systems with time delay. A modified L-K functional is developed, in particular, by Chunodkar and Akella[6] for spacecraft attitude stabilization with unknown but bounded delay in the feedback control loop. Exponential stability was obtained for all values of the time delay within the selected bounds.

Neural networks (NNs) are used to approximate unknown complex nonlinear functions in nonlinear dynamical systems with interconnection terms or when precise knowledge of the system dynamics is lacking. One main advantage of these schemes is that the adaptive laws are derived based on Lyapunov synthesis and, therefore, guarantee the stability of continuous-time systems without the requirement for offline training[7] . A NN approach is used by Yadmellat *et al*[8] for stabilization of nonlinear affine single-input systems, where a modified back propagation (BP) algorithm is employed to update the weights of the NN. Further, the second method of Lyapunov was implemented to investigate the stability of the system.

In this paper, the delayed feedback control of rigid spacecraft attitude dynamics is studied. Processing delays including time delays between the measurement and application of the control law would cause the time delay to appear in the feedback control. Here, the time delay is considered to exist in the sensors, while another method is investigated by the authors for the case where delay appears in the actuators[9] . The behavior of the system under two different control laws is investigated. First, the delayed states are fed to the NN for the control process. Next, another simulation is performed based on the velocity independent controller introduced by Ailon *et al*[1] . Unlike that study, and based on the descriptions given above on the advantages of using MRPs over other coordinate sets, however, the MRPs are used here as the attitude coordinates of the system. The closed-loop delayed system is integrated using MATLAB `dde23`. Although stability of the system depends on the NN error, by picking a suitable coefficient for the additional robust term in the feedback control, asymptotic stability of the closed loop system is guaranteed.

## PRELIMINARIES AND SYSTEM MODELING

The MRP vector, $\sigma \in \mathcal{R}^3$, can be expressed in terms of principle rotation elements as[2]

$$\vec{\sigma} = \tan\frac{\phi}{4}\,\hat{e}, \tag{1}$$

where $\hat{e}$ is the unit eigenvector of the rotation matrix $C$ corresponding to the eigenvalue of +1, $\phi$ is the principle rotation angle, and the matrix $C$ is the rotation matrix from the body frame to the inertial frame. More details about the attitude coordinate parameters are given in the literature[9,2] .

Consider the attitude dynamics of a rigid spacecraft as

$$\begin{aligned} \dot{\vec{\sigma}}(t) = &\quad \frac{1}{4}B(\vec{\sigma}(t))\vec{\omega}(t) \\ \dot{\vec{\omega}}(t) = &\quad -J^{-1}\tilde{\omega}(t)J\vec{\omega}(t) + J^{-1}\vec{u}(t), \end{aligned} \tag{2}$$

where $\vec{\omega}(t) \in \mathcal{R}^3$ is the angular velocity and $\vec{u}(t) \in \mathcal{R}^3$ is the control input analogous to the torque vector, $J \in \mathcal{R}^3$ is the inertia matrix, $\tilde{\omega}$ is defined as

$$\tilde{\omega} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}, \tag{3}$$

and the nonlinear matrix $B$ is defined as

$$B(\vec{\sigma}) = \left[\left(1 - \vec{\sigma}^T\vec{\sigma}\right) I_3 + 2\tilde{\sigma} + 2\vec{\sigma}\vec{\sigma}^T\right], \tag{4}$$

where $I_3$ is the three dimensional identity matrix.

By adding and subtracting $\frac{1}{4}\vec{\omega}(t)$ and $4\vec{\sigma}(t)$ to the first and the second parts of Eq. (2), respectively, it can be split into linear and (almost) nonlinear parts as

$$\begin{aligned}\dot{\vec{\sigma}}(t) = & \quad \frac{1}{4}\vec{\omega}(t) + \frac{1}{4}[B(\vec{\sigma}(t)) - I_3]\vec{\omega}(t), \\ \dot{\vec{\omega}}(t) = & \quad -4\vec{\sigma}(t) + \left[4\vec{\sigma}(t) - J^{-1}\tilde{\omega}(t)J\vec{\omega}(t)\right] + J^{-1}\vec{u}(t). \end{aligned} \tag{5}$$

Such an approach was also recently used in the delayed attitude control methodology suggested by Chunodkar and Akella.[6] Note that in Eqs. (2) and (5), the dynamics of the sensors and actuators have not been considered and it is supposed that three actuators are employed to provide the required torque about the body frame axes.

The control objective is to find a control law to regulate the system about an equilibrium point (the origin) in the presence of an unknown single discrete constant time delay in the measurements such that all the MRPs and angular velocities go to zero in some region of the state space as $t \to \infty$, i.e.

$$\lim_{t\to\infty} ||\vec{\sigma}|| = 0, \;\; \lim_{t\to\infty} ||\vec{\omega}|| = 0, \tag{6}$$

## DELAYED FEEDBACK CONTROLLER DESIGN VIA NEURAL NETWORKS

In this section, an adaptive NN technique is implemented to approximate the kinematic and inertial nonlinearities of the rigid body system, and subsequently a Lyapunov-Krasovskii functional is used to guarantee the local asymptotic stability of the otherwise linear closed loop delay differential system.

### Controller Design

It is assumed that the current states are unavailable for feedback due to measurement delays as shown in Fig. 1. The control law is therefore defined in the following form

$$\vec{u}(t) = -K_1\vec{\sigma}(t-\tau) - K_2\vec{\omega}(t-\tau) + \vec{\nu}(t) + \vec{a}(t), \tag{7}$$

where $\vec{\nu}(t)$ is a new control variable, $\vec{a}(t)$ is an additional robust term, $K_{1,2}$ are constant controller gain matrices with suitable dimensions, and $\tau$ is the known constant time delay in the feedback control. The schematic block diagram of the system is shown in Fig. 1, in which the delay is considered to be in the measurements.

Substituting Eq. (7) into Eq. (5) the closed loop dynamics can be obtained as

$$\begin{aligned}\dot{\vec{\sigma}}(t) = & \quad \frac{1}{4}\vec{\omega}(t) + \frac{1}{4}\left[B(\vec{\sigma}(t)) - I_3\right]\vec{\omega}(t) \\ \dot{\vec{\omega}}(t) = & \quad -4\vec{\sigma}(t) + \left[4\vec{\sigma}(t) - J^{-1}\tilde{\omega}(t)J\vec{\omega}(t)\right] - J^{-1}K_1\vec{\sigma}(t-\tau) - J^{-1}K_2\vec{\omega}(t-\tau) \\ & \quad +J^{-1}\vec{\nu}(t) + J^{-1}\vec{a}(t). \end{aligned} \tag{8}$$

The system above can be expressed in the form

$$\dot{\vec{x}}(t) = A\vec{x}(t) + A_d\vec{x}(t-\tau) + C(\vec{\nu}(t) + \vec{a}(t)) + \vec{f}(\vec{x}(t)), \tag{9}$$

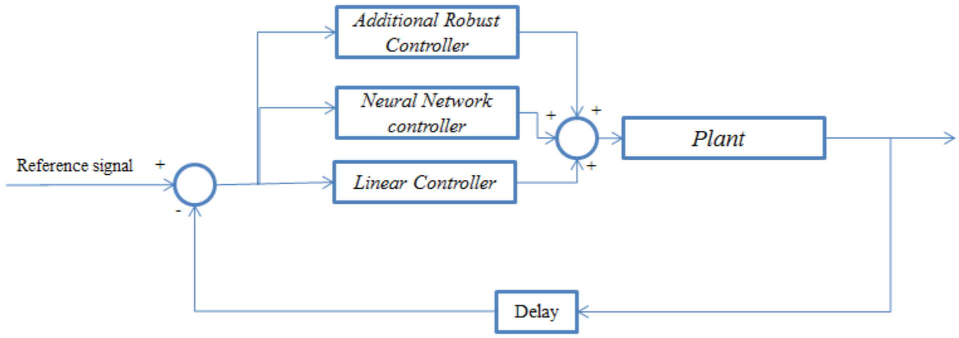


**Figure 1.** **Feedback control system with delay in the measurement, block diagram of the closed-loop system.**

where $\vec{x} = \begin{pmatrix} \vec{\sigma}^T & \vec{\omega}^T \end{pmatrix}^T \in \mathcal{R}^6$ is the state space vector, and

$$A = \begin{pmatrix} 0_3 & \frac{1}{4}I_3 \\ -4I_3 & 0_3 \end{pmatrix}, \; A_d = \bar{J}^{-1}\begin{pmatrix} 0_3 & 0_3 \\ -K_1 I_3 & -K_2 I_3 \end{pmatrix},$$
$$C = \bar{J}^{-1}\begin{pmatrix} 0_3 \\ I_3 \end{pmatrix},$$
$$\vec{f}(\vec{x}(t)) = \bar{J}^{-1}\begin{pmatrix} \frac{1}{4}\left(B(\vec{\sigma}(t)) - I_3\right)\vec{\omega}(t) \\ 4J\vec{\sigma}(t) - \tilde{\omega}(t)J\vec{\omega}(t) \end{pmatrix}. \tag{10}$$

The matrix $\bar{J}$ in Eq. (10) is defined as

$$\bar{J} = \begin{pmatrix} I_3 & 0_3 \\ 0_3 & J \end{pmatrix}. \tag{11}$$

### Neural Network Approximation

As discussed earlier, the time is assumed to be in the measurement, consequently, all the state variables are measured with time delay. Hence, implementing the feedback linearization method to cancel the (almost) purely nonlinear term $\vec{f}(\vec{x})$ in Eq. (9) is difficult. Therefore, we choose to approximate this unknown nonlinear vector-valued function by a feedforward NN on a compact set $S$ by the radial basis function

$$\vec{f}(\vec{x}(t)) = W\vec{\Phi}(\mathcal{V}\vec{x}) + \vec{\varepsilon}(t), \tag{12}$$

where $\mathcal{V}$ and $W$ are unknown constant weight matrices, $\vec{\Phi}(\mathcal{V}\vec{x}) : \; S \to \mathcal{R}^n$ is a known function with the NN node number $n > 1$, and $\vec{\varepsilon}(t)$ is the bounded NN approximation error (i.e. $||\vec{\varepsilon}|| \leq \varepsilon_N$ where $\varepsilon_N$ is a positive constant). The basis function $\vec{\Phi}(\vec{x})$ can be defined as a sigmoid function

$$\Phi_i(\mathcal{V}_i\vec{x}) = \frac{2}{1 + e^{-2\mathcal{V}_i\vec{x}}} - 1, \tag{13}$$

where $\mathcal{V}_i$ is the $i-$th row of the matrix $\mathcal{V}$, and $\Phi_i(\mathcal{V}_i\vec{x})$ is the $i-$th element of the vector $\vec{\Phi}(\mathcal{V}\vec{x})$. For simplicity purposes, the weight matrix $\mathcal{V}$ is considered to be the $n$ dimensional identity matrix, and the optimal weight matrix $W$ is defined as

$$W = \arg \min_{\hat{W} \in \mathcal{R}^n} \left\{ \sup_{x \in S} \left| \vec{f}(\vec{x}) - \hat{W}(t)\Phi(\vec{x}) \right| \right\}, \tag{14}$$

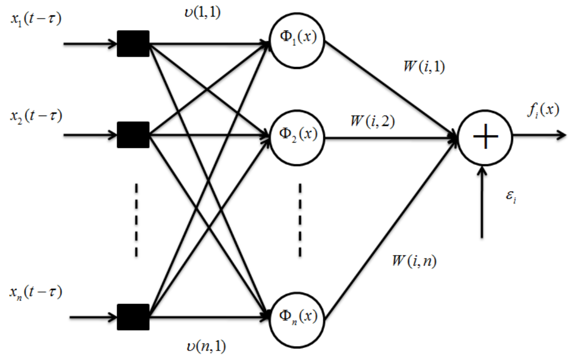


**Figure 2. Neural Network Scheme**

where $\hat{W}(t)$ represents the estimated weight matrix. The structure of $i-$th $(i = 1, 2, \cdots, n)$ branch of the NN is illustrated in Fig. 2. Using the above neural network approximation along with the control law in Eq. (7), we can obtain the following result which guarantees the local stability of the system.

**Remark.** *Note that the MRPs are used based on the advantages mentioned before, and, to the knowledge of the authors, the combination of MRPs and the proposed NN does not necessarily provide any benefits over the other attitude parameterizations beyond these well-known advantages of MRPs.*

**Theorem 1** *Consider the system described by Eq. (5) and the control law given in Eq. (7) with the new control variable*

$$\vec{\nu}(t) = -C^{+}\hat{W}(t)\vec{\Phi}(\vec{x}), \tag{15}$$

*where $C^{+}$ denotes the pseudoinverse of the matrix $C$ such that*

$$CC^{+} = I_6, \tag{16}$$

*where $I_6$ is the $6 \times 6$ identity matrix, and*

$$\vec{a}(t) = C^{+}\kappa\vec{x}(t-\tau), \tag{17}$$

*is an additional robust term, where $\kappa$ is a constant. If the weights of the NNs are updated according to*

$$\dot{\hat{W}}(t) = -\eta\frac{\partial\mathcal{J}}{\partial\hat{W}(t)} \tag{18}$$

*where $\eta > 0$ is an arbitrary constant, and $\mathcal{J}$ is the cost function defined as*

$$\mathcal{J} = \frac{1}{2}\vec{x}^{T}(t)\vec{x}(t), \tag{19}$$

*then asymptotic stability of the system is guaranteed if there exists positive definite matrices $P$, $Q$, $K_1$, and $K_2$ such that the following linear matrix inequality (LMI) holds*

$$M = \begin{pmatrix} A^{T}P + PA + Q & PA_d & F^{T} + P \\ {A_d}^{T}P & -Q & 0_3 \\ F + P & 0_3 & 0_3 \end{pmatrix} \leq 0, \tag{20}$$

*where $F = \eta\Gamma {A^{-1}}^T$ and $\Gamma$ is a constant symmetric positive definite matrix. Thus all the signals in the closed loop system are bounded which shows the local stability condition for the system.*

***Proof 1*** *Defining the weighted matrix error as*

$$\tilde{W}(t) = W - \hat{W}(t) \tag{21}$$

*the Lyapunov-Krasovskii functional candidate is assumed as*

$$\begin{aligned} V(t) = \quad & \vec{x}^T(t)P\vec{x}(t) + \\ & \int_{t-\tau}^{t} \vec{x}^T(\gamma)Q\vec{x}(\gamma)d\gamma + \mathrm{tr}(\tilde{W}(t)^T \Gamma \tilde{W}(t)), \end{aligned} \tag{22}$$

*where $P$, $Q$, and $\Gamma$ are constant symmetric positive definite matrices with suitable dimensions. Taking the time derivative of $V(t)$ along the trajectory of the closed loop dynamical system we obtain*

$$\begin{aligned} \dot{V}(t) = \quad & \dot{\vec{x}}^T(t)P\vec{x}(t) + \vec{x}^T(t)P\dot{\vec{x}}(t) + \vec{x}^T(t)Q\vec{x}(t) - \\ & \vec{x}^T(t-\tau)Q\vec{x}(t-\tau) + 2\mathrm{tr}(\tilde{W}^T(t)\Gamma\dot{\tilde{W}}(t)). \end{aligned} \tag{23}$$

*Substituting Eqs. (12), (15), and (17) into (9), and plugging the result into into (23) we obtain*

$$\begin{aligned} \dot{V}(t) = \quad & \vec{x}^T(t)(A^TP + PA + Q)\vec{x}(t) + \vec{x}^T(t-\tau)(-Q)\vec{x}(t-\tau) \\ & +\vec{x}^T(t)PA_d\vec{x}(t-\tau) + \vec{x}^T(t-\tau){A_d}^TP\vec{x}(t) + \\ & \vec{\Phi}^T(\vec{x})\tilde{W}^TP\vec{x}(t) + \vec{\varepsilon}^T(t)P\vec{x}(t) + \kappa\vec{x}^T(t-\tau)P\vec{x}(t) + \\ & \vec{x}^T(t)P\tilde{W}\vec{\Phi}(\vec{x}) + \vec{x}^T(t)P\vec{\varepsilon}(t) + \kappa\vec{x}^T(t)P\vec{x}(t-\tau) + \\ & 2\mathrm{tr}(\tilde{W}^T(t)\Gamma\dot{\tilde{W}}(t)). \end{aligned} \tag{24}$$

*The adaptive control law $\hat{W}(t)$ can be calculated by using the BP algorithm as[8] in Eq. (18).*

*On the other hand, the cost function defined in Eq. (19) and the control variable given in Eq. (15) are used in the following chain rule to obtain the trace of the partial derivative of the cost function with respect to the estimated weights vector as*

$$\mathrm{tr}\left(\frac{\partial \mathcal{J}}{\partial \hat{W}}\right) = \mathrm{tr}\left(\frac{\partial \mathcal{J}}{\partial \vec{x}}\frac{\partial \vec{x}}{\partial \vec{\nu}}\frac{\partial \vec{\nu}}{\partial \hat{W}}\right) = \vec{x}^T\left(\frac{\partial \vec{x}}{\partial \hat{\nu}}\right)\left(-C^+\vec{\Phi}(\vec{x})\right). \tag{25}$$

*In order to calculate the term $\frac{\partial \vec{x}}{\partial \hat{\nu}}$ in Eq. (25) the same strategy is used as in the literature[8, 10], where the static approximation gradient $\dot{\vec{x}} \approx \vec{0}$ is assumed. Hence from Eq. (9)*

$$\frac{\partial \vec{x}}{\partial \hat{\nu}} = -A^{-1}C. \tag{26}$$

*Inserting Eq. (26) into Eq. (25), and using the property (16) we obtain*

$$\mathrm{tr}\left(\frac{\partial \mathcal{J}}{\partial \hat{W}}\right) = \underbrace{\vec{x}^T A^{-1}}_{\vec{b}_1^T}\underbrace{\vec{\Phi}(\vec{x})}_{\vec{b}_2}. \tag{27}$$

*By using the identity* $\mathrm{tr}(\vec{b}_1\vec{b}_2^T) = \vec{b}_1^T\vec{b}_2$*, Eq. (27) yields*

$$\frac{\partial \mathcal{J}}{\partial \hat{W}} = \left(\vec{x}^T A^{-1}\right)^T \vec{\Phi}^T(\vec{x}). \tag{28}$$

*Now, by substituting the result back into Eqs. (18) and (21) the NN learning rule can be approximated as*

$$\dot{\tilde{W}}(t) = -\dot{\hat{W}}(t) = \eta \left(\vec{x}^T A^{-1}\right)^T \vec{\Phi}^T(\vec{x}). \tag{29}$$

*Hence, the term* $2\mathrm{tr}(\tilde{W}^T\Gamma\dot{\tilde{W}})$ *in Eq. (24) can be written as*

$$2\mathrm{tr}\left(\tilde{W}^T\Gamma\eta \left(\vec{x}^T A^{-1}\right)^T \vec{\Phi}^T(\vec{x})\right). \tag{30}$$

*Defining* $F$ *as* $F = \eta\Gamma A^{-1^T}$*, Eq. (30) becomes*

$$\begin{aligned} 2\mathrm{tr}\left(\tilde{W}^T F\vec{x}\vec{\Phi}^T(\vec{x})\right) = 2\vec{\Phi}^T\tilde{W}^T F\vec{x} = \quad & \vec{\Phi}^T\tilde{W}^T F\vec{x} + \\ & \vec{x}^T F^T\tilde{W}\vec{\Phi}. \end{aligned} \tag{31}$$

*Therefore,* $\dot{V}(t)$ *in Eq. (24) can be represented as*

$$\begin{aligned} \dot{V}(t) = \quad & \vec{x}^T(t)(A^TP + PA + Q)\vec{x}(t) + \vec{x}^T(t-\tau)(-Q)\vec{x}(t-\tau) + \\ & \vec{x}^T(t)PA_d\vec{x}(t-\tau) + \vec{x}^T(t-\tau)(A_d{}^TP)\vec{x}(t) + \\ & \vec{\Phi}^T(\vec{x})\tilde{W}^T(F+P)\vec{x}(t) + \vec{x}^T(t)(F^T+P)\tilde{W}\vec{\Phi}(\vec{x}) + \\ & (\kappa\vec{x}(t-\tau) + \vec{\varepsilon}(t))^T P\vec{x}(t) + \vec{x}^T(t)P(\vec{\varepsilon}(t) + \kappa\vec{x}(t-\tau)) \end{aligned} \tag{32}$$

*The augmented vector* $\vec{\xi}$ *is defined as*

$$\vec{\xi} = \left\{ \begin{array}{c} \vec{x}(t) \\ \vec{x}(t-\tau) \\ \tilde{W}\vec{\Phi}(\vec{x}) \end{array} \right\} \tag{33}$$

*such that Eq. (32) becomes*

$$\begin{aligned} \dot{V}(t) = \quad & \vec{\xi}^T\mathcal{M}\vec{\xi} + (\vec{\varepsilon} + \kappa\vec{x}(t-\tau))^T P\vec{x}(t) + \vec{x}^T(t)P\left(\vec{\varepsilon} + \kappa\vec{x}(t-\tau)\right) \\ = \quad & \vec{\xi}^T\mathcal{M}\vec{\xi} + 2\vec{x}^T(t)P\left(\vec{\varepsilon} + \kappa\vec{x}(t-\tau)\right), \end{aligned} \tag{34}$$

*where*

$$\mathcal{M} = \begin{pmatrix} A^TP + PA + Q & PA_d & F^T + P \\ A_d{}^TP & -Q & 0_3 \\ F + P & 0_3 & 0_3 \end{pmatrix} \tag{35}$$

*can be investigated using standard LMI theory. In order to guarantee that the last two terms in the right hand side of Eq. (34) are negative, we set*

$$\vec{\varepsilon}(t) + \kappa\vec{x}(t-\tau) = -\alpha\vec{x}(t), \;\; \alpha > 0. \tag{36}$$

*For a stable system $||\vec{x}(t)|| \leq ||\vec{x}(t-\tau)||$ as t goes to infinity, hence Eq. (36) implies that*

$$\begin{aligned}\kappa = & -\frac{\vec{x}^T(t-\tau)\vec{\varepsilon}(t) + \alpha\vec{x}^T(t-\tau)\vec{x}(t)}{||\vec{x}(t-\tau)||^2} \\ = & -\frac{\vec{x}^T(t-\tau)\vec{\varepsilon}(t)}{||\vec{x}(t-\tau)||^2} - \alpha\frac{\vec{x}^T(t-\tau)\vec{x}(t)}{||\vec{x}(t-\tau)||^2} \\ \leq & \frac{\varepsilon_N}{||\vec{x}(t-\tau)||} + \alpha \leq \frac{\varepsilon_N}{||\vec{x}(t)||} + \alpha.\end{aligned} \tag{37}$$

*For a proper value of $\kappa$ that satisfies Eq. (37), plugging Eq. (36) into (34) yields*

$$\dot{V}(t) = \vec{\xi}\mathcal{M}\vec{\xi} + \vec{x}^T(-2\alpha P)\vec{x} < 0. \tag{38}$$

*Since $\mathcal{M}$ and $-2\alpha P$ are negative definite matrices, one can say $\dot{V}(t) < 0$ occurs only at the equilibrium point of the system which guarantees asymptotic stability of the system in some regions by Lyapunov's second method, and the proof is complete.* □

## DELAYED FEEDBACK CONTROL VIA A VELOCITY-FREE APPROACH

The following technique adapted from[1] is used in this paper to compare the performance of the proposed neural network approach presented above. In the first case, for attitude regulation, the delay is considered to be known and $u$ is assumed as an input torque in terms of the state space parameters and is injected into the system at $t = 0$[1]

$$\vec{u} = -\frac{1}{4}B^T(\vec{\sigma})KN(\vec{\sigma} - \vec{z}), \tag{39}$$

where the meta-state $\vec{z}$ is such that

$$\dot{\vec{z}} = -(N + M)\vec{z} + N\vec{\sigma} + \vec{L}, \tag{40}$$

and where $\vec{L}$ is a constant feedback vector whose value depends on the selected set-point. Thus, if we set $\vec{\sigma} = [0, 0, 0]^T$, then $\vec{L}$ will be zero. The $3 \times 3$ constant symmetric matrices $N$, $M$, and $K$ should be chosen such that they have the commutative property

$$KN = NK, \quad KM = MK, \quad MN = NM. \tag{41}$$

In this method, a torque is applied to control the system which can be either due to the thrusts or the change of moments about the gyroscopic axes.

### Stability Analysis of the Corresponding Delay Free System

Let us first consider the delay free case. In order to prove that the origin of Eqs. (2), (39), and (40) is asymptotically stable, consider the following Lyapunov candidate function[1]

$$V(x) = \frac{1}{2}\left[\vec{\omega}^T J\vec{\omega} + (\vec{\sigma} - \vec{z})^T KN(\vec{\sigma} - \vec{z}) + \vec{z}^T KM\vec{z}\right] > 0. \tag{42}$$

The time derivative is found as

$$
\begin{aligned}
\dot{V}(\vec{x}) = \quad & \dot{\vec{\omega}}^T J\vec{\omega}_(\dot{\vec{\sigma}} - \dot{\vec{z}})^T KN(\vec{\sigma} - \vec{z}) + \dot{\vec{z}}^T KM\vec{z} \\
= \quad & \left[J^{-1}\tilde{\omega}J\vec{\omega} - J^{-1}B^T(\vec{\sigma})KN(\vec{\sigma} - \vec{z})\right]^T J\vec{\omega} + \left[\frac{1}{4}B(\vec{\sigma})\vec{\omega} + (N+M)\vec{z} - N\vec{\sigma}\right]^T KN(\vec{\sigma} - \vec{z}) + \\
& \left[-(N+M)\vec{z} + N\vec{\sigma}\right]^T KM\vec{z} \\
= \quad & \left[-\vec{\omega}^T J\tilde{\omega}J^{-1} - (\vec{\sigma}^T - \vec{z}^T)KN\frac{1}{4}B(\vec{\sigma})J^{-1}\right] J\vec{\omega} + \\
& \left[\vec{\omega}^T \frac{1}{4}B^T(\vec{\sigma}) + \vec{z}^T(N+M) - \vec{\sigma}^T N\right] KN(\vec{\sigma} - \vec{z}) + \left[-\vec{z}^T(N+M) + \vec{\sigma}^T N\right] KM\vec{z} \\
= \quad & -\vec{\omega}^T J\tilde{\omega}\vec{\omega} - (\vec{\sigma}^T - \vec{z}^T)KN\frac{1}{2}\left(I_3 - \tilde{\sigma} + \vec{\sigma}\vec{\sigma}^T\right)\vec{\omega} + \\
& \left[\vec{\omega}^T \frac{1}{2}(I_3 + \tilde{\sigma} + \vec{\sigma}\vec{\sigma}^T) + \vec{z}^T(N+M) - \vec{\sigma}^T N\right] KN(\vec{\sigma} - \vec{z}) + \\
& \left[-\vec{z}(N+M) + \vec{\sigma}^T N\right] KM\vec{z}.
\end{aligned} \tag{43}
$$

However, $\tilde{\omega}\vec{\omega} = \vec{\omega} \times \vec{\omega} = 0$, and using the facts that $KN\vec{\sigma}\vec{\sigma}^T = \vec{\sigma}\vec{\sigma}^T KN$, and that for vectors $\vec{\alpha}, \vec{\beta}$ and matrix $\mathcal{A}$, $\vec{\alpha}^T\mathcal{A}\vec{\beta} = \beta^T\mathcal{A}\vec{\alpha}$, Eq. (43) will be

$$
\begin{aligned}
\dot{V}(\vec{x}) = \quad & -\frac{1}{2}\left[\vec{\omega}^T KN\vec{\sigma} - \vec{\omega}^T KN\vec{z} + \vec{\omega}^T KN\tilde{\sigma}\vec{\sigma} + \vec{\omega}^T KN\tilde{\sigma}\vec{z} + \vec{\omega}^T KN\vec{\sigma}\vec{\sigma}^T\vec{\sigma} - \vec{\omega}^T KN\vec{\sigma}\vec{\sigma}^T\vec{z}\right] + \\
& \left[\vec{\omega}^T\frac{1}{2}KN(\vec{\sigma} - \vec{z}) + \frac{1}{2}\vec{\omega}^T\tilde{\sigma}KN(\vec{\sigma} - \vec{z}) + \frac{1}{2}\vec{\omega}^T\vec{\sigma}\vec{\sigma}^T KN(\vec{\sigma} - \vec{z}) + \right. \\
& \left.\vec{z}^T(N+M)KN(\vec{\sigma} - \vec{z}) - \vec{\sigma}^T NKN(\vec{\sigma} - \vec{z})\right] + \left[-\vec{z}^T(N+M)KM\vec{z} + \vec{\sigma}^T NKM\vec{z}\right] \\
= \quad & -\vec{z}^T(N+M)K(N+M)\vec{z} + \vec{z}^T(N+M)KN\vec{\sigma} + \vec{\sigma}^T NK(N+M)\vec{z} - \vec{\sigma}^T NKN\vec{\sigma} \\
= \quad & \vec{z}^T(N+M)K\left[-(N+M)\vec{z} + N\vec{\sigma}\right] - \vec{\sigma}^T NK\left[-(N+M)\vec{z} + N\vec{\sigma}\right] \\
= \quad & \left[\vec{z}^T(N+M) - \vec{\sigma}^T N\right] K\left[-(N+M)\vec{z} + N\vec{\sigma}\right] = -\dot{\vec{z}}^T K\dot{\vec{z}} \le 0,
\end{aligned}
$$

which is negative semi definite. Now, applying the *LaSalle* invariance principle, the set $\mathcal{S}$ is defined as

$$\mathcal{S} = \left\{\vec{x} \in D | \dot{V}(\vec{x}) = 0\right\}. \tag{44}$$

Setting $\dot{V}$ equal to zero implies that

$$\dot{\vec{z}}(t) = \vec{0}, \quad \ddot{\vec{z}}(t) = \vec{0}. \tag{45}$$

Equation (2) reads

$$\ddot{\vec{z}}(t) = -(N+M)\dot{\vec{z}} + N\dot{\vec{\sigma}} = \vec{0}, \tag{46}$$

from which $\dot{\vec{\sigma}}(t) = 0$. Again, referring to Eq. (2), either $\vec{\omega}$ or $B(\vec{\sigma})$ must be zero. However, the diagonal arrays of the matrix $B(\vec{\sigma})$ are $1 + \sigma_i^2$ for $i = 1, 2, 3$, and thus, $B(\vec{\sigma})$ can never be zero, which along with previous conclusion reads $\vec{\omega}(t) = 0$ and consequently $\dot{\vec{\omega}}(t) = 0$, which the latter, according to Eq. (39) implies that $\vec{\sigma} = \vec{z}$. On the other hand and referring to Eq. (40), $\dot{\vec{z}} = -(N+M)\vec{z} + N\vec{\sigma} = -M\vec{z} = \vec{0}$. Therefore,

$$\vec{z} = \vec{\sigma} = \vec{\omega} = \vec{0}. \tag{47}$$

In other words, we have proved that no solution stays in the set $\mathcal{S}$ other than the origin, and thus, the origin is asymptotically stable for the delay free case.

In the calculations above,

$$\vec{x} = \left\{ \begin{array}{c} \vec{\sigma} \\ \vec{\omega} \\ \vec{z} \end{array} \right\} \in \Re^9 \tag{48}$$

is the assembled state vector. Note that all variables and parameters stated so far indicate vectors or matrices except for the $V$, $\dot{V}$, and, of course, the time $t$.

**Stability Analysis of the Delayed System**

Equations (2), (39), and (40) with delayed feedback can be written as ($\vec{L} = \vec{0}$)

$$\begin{aligned}
\dot{\vec{\sigma}} = &\quad \frac{1}{2}\vec{\omega} + \frac{1}{4}\bar{B}(\vec{\sigma})\vec{\omega}, \\
\dot{\vec{\omega}} = &\quad J^{-1}\tilde{\omega}J\vec{\omega} + J^{-1}\vec{u}(t-\tau) \\
= &\quad J^{-1}\tilde{\omega}J\vec{\omega} - \frac{1}{2}J^{-1}KN[\vec{\sigma}(t-\tau) - \vec{z}(t-\tau)] - J^{-1}\frac{1}{4}\bar{B}\left(\vec{\sigma}(t-\tau)\right)KN[\vec{\sigma}(t-\tau) - \vec{z}(t-\tau)] \\
\dot{\vec{z}} = &\quad -(N+M)\vec{z} + N\vec{\sigma}
\end{aligned} \tag{49}$$

for $t \geq 0$, where $\frac{1}{4}\bar{B} = -\frac{1}{2}I_3 + \frac{1}{4}B$, and $\vec{x}(t) = \vec{\phi}(t)$ for $t \in [-\tau, 0]$. Splitting the time invariant and time-varying matrix multiplier, the governing equations of the system (49) can be written as

$$\dot{\vec{x}} = E\vec{x}(t) + F\vec{x}(t-\tau) + G(\vec{x}(t))\vec{x}(t) + H(\vec{x}(t-\tau))\vec{x}(t-\tau), \tag{50}$$

where

$$\begin{aligned}
E &= \begin{pmatrix} 0_3 & \frac{1}{2}I_3 & 0_3 \\ 0_3 & 0_3 & 0_3 \\ N & 0_3 & -(N+M) \end{pmatrix}, \quad F = \begin{pmatrix} 0_3 & 0_3 & 0_3 \\ -\frac{1}{2}J^{-1}KN & 0_3 & \frac{1}{2}J^{-1}KN \\ 0_3 & 0_3 & 0_3 \end{pmatrix}, \\
G(\vec{x}(t)) &= \begin{pmatrix} 0_3 & \frac{1}{4}\bar{B}(\vec{\sigma}(t)) & 0_3 \\ 0_3 & J^{-1}\tilde{\omega}(t)J & 0_3 \\ 0_3 & 0_3 & 0_3 \end{pmatrix}, \\
H(\vec{x}(t-\tau)) &= \begin{pmatrix} 0_3 & 0_3 & 0_3 \\ -\frac{1}{4}J^{-1}\bar{B}^T\left(\vec{\sigma}(t-\tau)\right)KN & 0_3 & \frac{1}{4}J^{-1}\bar{B}^T\left(\vec{\sigma}(t-\tau)\right)KN \\ 0_3 & 0_3 & 0_3 \end{pmatrix},
\end{aligned} \tag{51}$$

and the matrices are separated based upon whether they are multipliers of $\vec{x}(t)$ and $\vec{x}(t-\tau)$, and upon whether they are constant or time-varying. The response of the system can then be obtained by using the convolution integral (method of steps) as

$$\begin{aligned}
\vec{x}(t) = \exp(Et)\vec{\phi}(0) + \int_0^t &\quad \exp\left(E.(t-\tau_1)\right)\Big[F\vec{\phi}(\tau_1-\tau) + G(x(\tau_1))\vec{x}(\tau_1) + \\
&\quad H\left(\vec{\phi}(\tau_1-\tau)\right)\vec{\phi}(\tau_1-\tau)\Big]\,d\tau_1, \quad t \in [0,\tau]
\end{aligned} \tag{52}$$

$$\vec{x}(t) = \exp\left((E+F).(t-\tau)\right)\vec{x}(\tau) + \int_{\tau}^{t} \exp\left((E+F).(t-\tau)\right)\left\{\int_{\tau_1-\tau}^{\tau_1}\left[-FE\vec{x} - FG(\vec{x})\vec{x}\right]d\tau_2 + G(\vec{x})\vec{x} + H(\vec{x}(\tau_1-\tau))\vec{x}(\tau_1-\tau)\right\}d\tau_1, \quad t \geq \tau \tag{53}$$

or by numerical integration.

**Proposition 1** *A set of positive constants $a_f$ , $a_e$, $a_{fe}$, $\alpha$, and $k, \zeta \geq 1$ can be selected such that*[1]

$$a_f = ||F||,\ a_e = ||E||,\ a_{fe} = ||FE|| > 0, \quad ||\exp(At)|| \leq k\exp(-\alpha t),\ t \geq 0$$
$$\sup_{t\in[0,\tau]} ||\exp(Et)|| \leq \zeta \tag{54}$$

**Proof 2** *See reference*[1] . □

**Theorem 2** *Considering Eq. (54), if one defines q as*

$$q = \frac{ka_{fe}}{\alpha}\tau, \tag{55}$$

*then for $q < 1$, the trivial solution of the system given in (50) is locally but exponentially stable.*

**Proof 3** *See reference*[1] . □

## SIMULATION RESULTS

In this section, the proposed NN-based controller is first implemented and its results are compared to the velocity-free controller introduced in Section . In order to develop the simulations based on the velocity-free controller, the feedback controller in Eq. (39) is implemented to the system and MATLAB `dde23` is used in the simulations to integrate the time delayed system. The syntax of `dde23` is given by

```
sol = dde23(ddefun,lags,history,tspan)
```
(56)

where `ddefun` is the handle to the function describing the system, `lags` is a vector of discrete delays, `history` is the handle describing the initial function, and `tspan` is the vector of the integration limits. Finally, the norm of the assembled state-space vector ($||\vec{x}|| = \sqrt{||\vec{\sigma}||^2 + ||\vec{\omega}||^2 + ||\vec{z}||^2}$), and the variation of the feedback control law $\vec{u}(t)$ are obtained versus time, and compared to those obtained in the literature.

In the first step, in order to compare the proposed NN-based controller with the velocity-free controller, the system parameters for two different cases given in Table 1 are considered. Figures 3–6 compare the results obtained by our NN-based controller to those obtained using velocity-free controller for a small spacecraft (case 1) and a large one (case 2) (see Table 1) with the initial conditions $\vec{\sigma}_0 = [-0.3, -0.4, 0.2]^T$ (rad) and $\vec{\omega}_0 = [0.2, 0.2, 0.2]^T$ $\left(\frac{\text{rad}}{\text{sec}}\right)$ for two different values of the time delay ($\tau = 0.1$ sec and $\tau = 0.5$ sec). It should be mentioned here that the initial meta-state vector $\vec{z}$ for the velocity-free controller is zero in both cases. The constant control parameters $\eta$ and $\kappa$ for the NN-based controller are also given in Table 1.

**Table 1. System parameters for two sample cases**

| Parameter | Case 1 | Case 2 |
|---|---|---|
| $J$ $\left(\mathrm{Kg.m}^2\right)$ | $\mathrm{diag}(50, 30, 20)$ | $\mathrm{diag}(1000, 700, 500)$ |
| $\tau$ (sec) | 0.1, 0.5 | 0.1, 0.5 |
| $\eta$ | 0.01 | 7 |
| $\kappa$ | -75 | -1575 |

**Table 2. Parameter values as defined by Ailon *et al*[1]**

| Parameter Matrix | Value |
|---|---|
| $J$ | $\mathrm{diag}\begin{bmatrix} 1000 & 700 & 500 \end{bmatrix}$ |
| $K$ | $\mathrm{diag}\begin{bmatrix} 1035 & 517.5 & 724.5 \end{bmatrix}$ |
| $M$ | $\mathrm{diag}\begin{bmatrix} .0767 & .0767 & .0767 \end{bmatrix}$ |
| $N$ | $\mathrm{diag}\begin{bmatrix} .6128 & .6128 & .6128 \end{bmatrix}$ |

As shown in the figures, the magnitude of the controlled signal in the NN approach is initially higher than those obtained using the approach in Eqs. (39) and (40). The reason is that all the nonlinear terms in the present approach are supposed to be unknown and must be approximated. Therefore, it is reasonable for the magnitude of the proposed controller to be higher than that in Eqs. (39) and (40). Furthermore, there are no considerable over- or undershoots in the responses obtained by the NN approximation, whereas all the controlled states in the velocity-free controller approach have some over- or undershoots before they die out. Since the values of the state variables at the current time are unknown, delayed states are considered in the NN-based controller function (7) and the NNs are subject to some approximated *nonlinear* terms with the time delay in the input as well as delayed linear terms. Another advantage of the NN-based controller is its delay-independence which means that for proper controller gains $\eta$ and $\kappa$, as long as there exist positive definite matrices $P$, $Q$, $K_1$, and $K_2$ such that the LMI (20) holds, the proposed delayed feedback controller can regulate the system, no matter what the delay value is. It should be mentioned that in order to regulate the system with higher values of the time delay, the control gains $\kappa$ and $\eta$ should be changed accordingly which may result in higher torque magnitude and consequently higher control cost which may not be practical. The results obtained by the NN-based controller are compared to those obtained by velocity-free controller in Fig. 7 for the large spacecraft with $\tau = 1.1$ sec, where the controller gains in the NN-based controller are $\kappa = -1100$, and $\eta = 700$. As can be seen in the figure, the system is controlled by NN-based controller while the velocity-free controller cannot control the system for this time delay. By performing some simulations we found that the maximum value of the time delay which can be handled by the velocity-free controller is about $\tau \approx 1.085$ sec.

Further, we consider the same inertia matrix $J$, and the symmetric gain matrices $K$, $M$ and $N$ (Table 2) as those given by Ailon *et al*[1] . The results for the stable system with time delay $\tau = 0.0125$ and initial state space vector $\vec{\phi}(\theta) = [0, 0.001, -0.001, 0, 0, 0, 0.001, 0, 0]^T$ for $\theta \in [-\tau, 0]$ are compared to those obtained by Ailon *et al*[1] in Fig. 8. Differences with the results can be attributed to the fact that Ailon *et al*[1] have used CRPs for their simulation, while here MRPs are used instead.

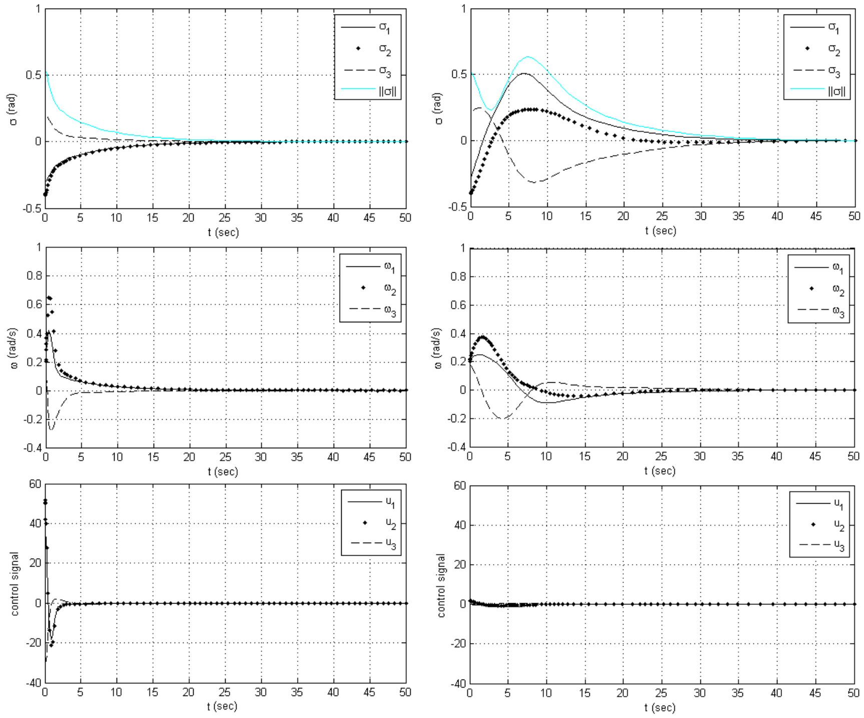


**Figure 3. Neural network approach (Left) vs. velocity-free controller (Right) for the small spacecraft (see Table 1) with $\tau = 0.1$ sec.**

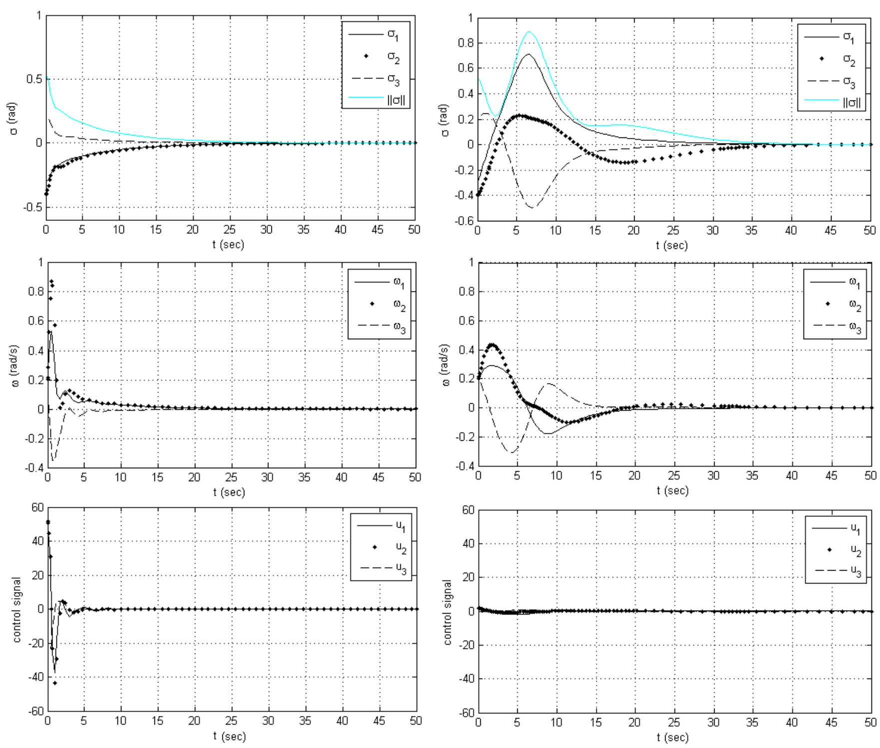


**Figure 4. Neural network approach (Left) vs. velocity-free controller (Right) for the small spacecraft (see Table 1) with $\tau = 0.5$ sec.**

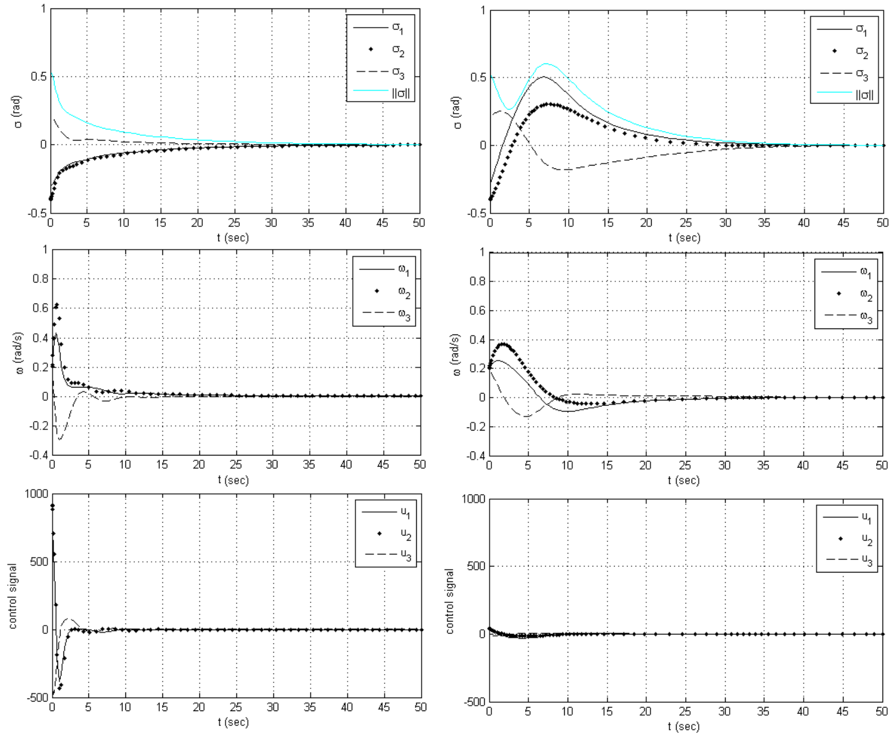


**Figure 5. Neural network approach (Left) vs. velocity-free controller (Right) for the large spacecraft (see Table 1) with $\tau = 0.1$ sec.**

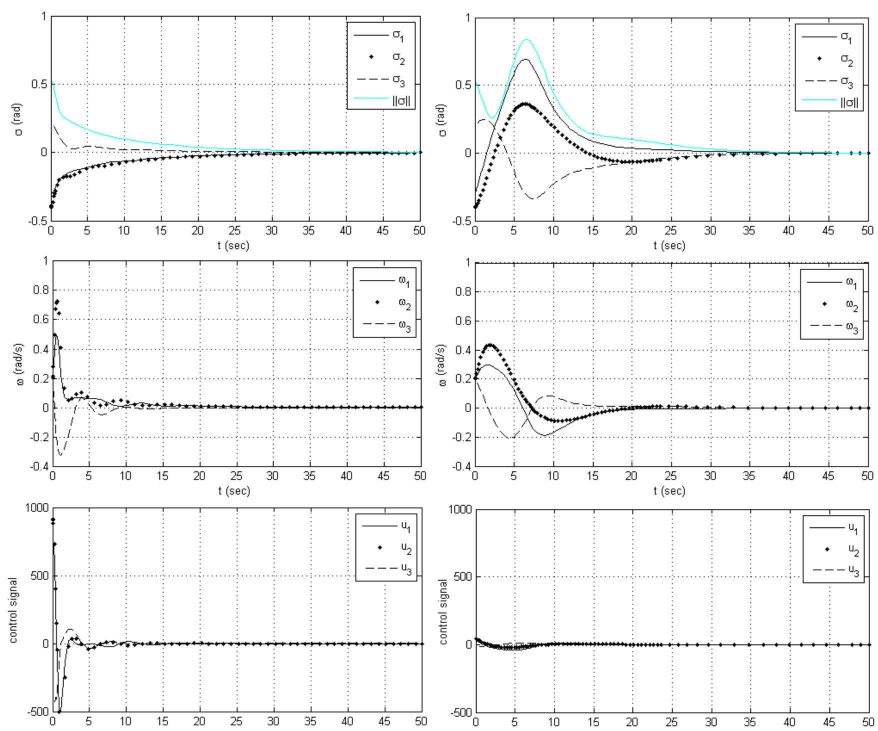


**Figure 6. Neural network approach (Left) vs. velocity-free controller (Right) for the large spacecraft (see Table 1) with $\tau = 0.5$ sec.**

## CONCLUSIONS

The delayed feedback control of spacecraft attitude dynamics is studied in this paper where an unknown discrete time delay is considered in the feedback control of a rigid spacecraft, and the MRP vector is selected as the attitude coordinate set. The advantage of using MRPs is that they can describe tumbling motion by switching to the shadow set at a certain value of principle rotation angle. A NN approximation along with a suitable Lyapunov-Krasovskii functional have been implemented to investigate the regulation of the closed-loop controlled system. Finally, the results of the NN approach are compared to those of a simulation based on the velocity-independent controller given in the literature[1] . The proposed controller has shown a smoother response in terms of the over- and undershoots for the controlled states as compared to the velocity-free approach, and can guarantee the local stability of the closed-loop system. According to the results of the NN method, and due to the delay-independence, even for higher values of time delay the local stability of the system is guaranteed with proper selection of the control gains. A possible future work in this area would be investigating the effectiveness of the proposed controller on tumbling motion.

## ACKNOWLEDGMENTS

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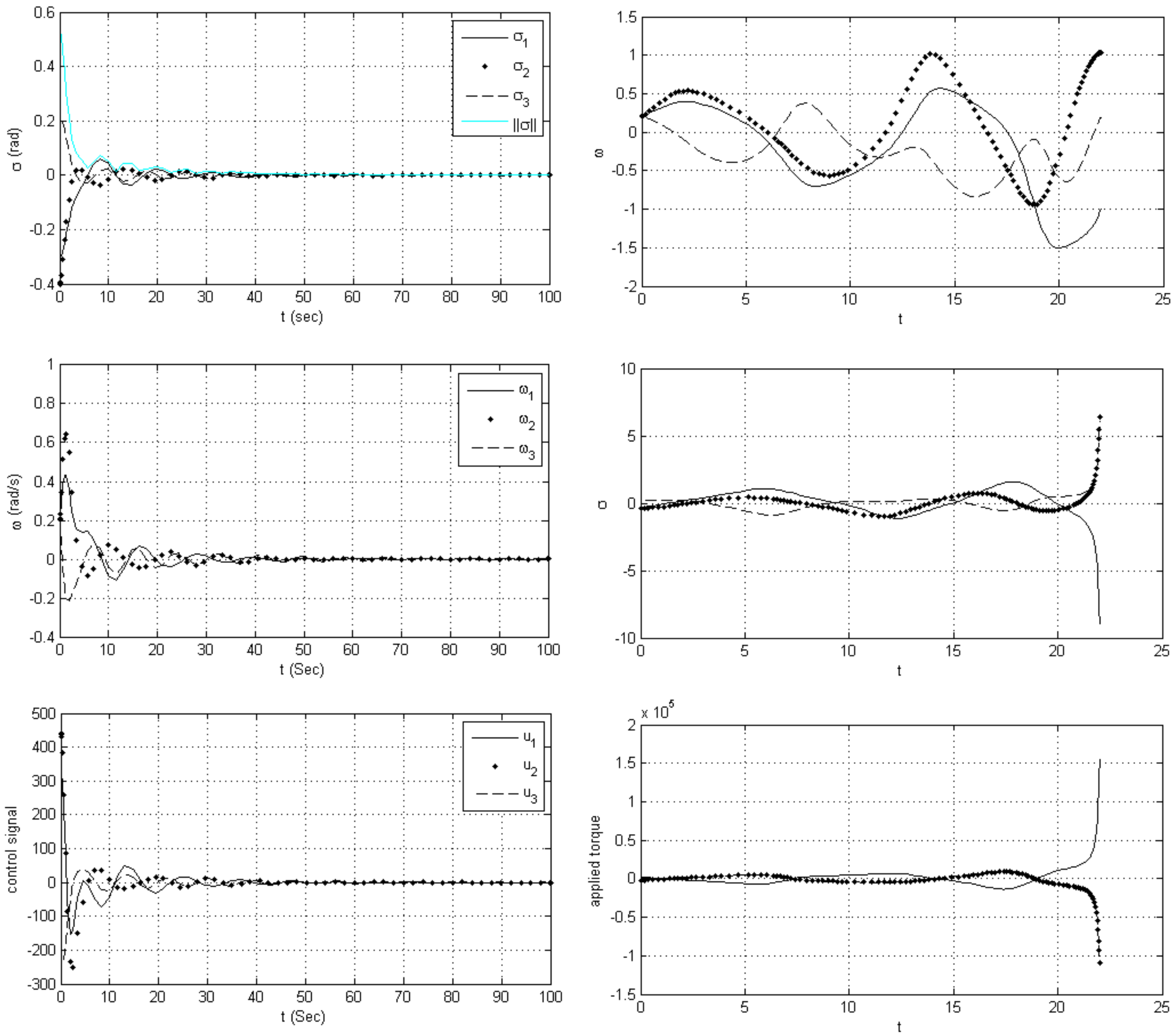


**Figure 7. Neural network approach (Left) vs. velocity-free controller (Right) for the large spacecraft (see Table 1) with $\tau = 1.1$ sec.**

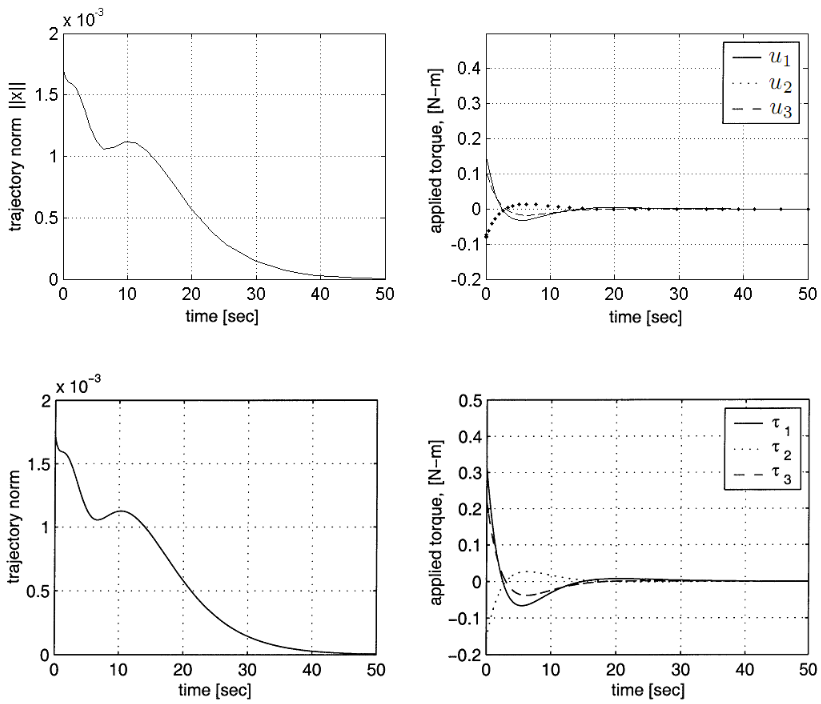


**Figure 8. Simulation results for the stable case ($\tau = 0.0125$) based on the velocity-free controller; present study with the use of MRPs (Top); results given in Ref.[1] with the use of CRPs (Bottom)**