EFFECTS OF CHARGED DIELECTRICS ON ELECTROSTATIC FORCE AND TORQUE

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Charged spacecraft experience electrostatic forces and torques from both charged neighboring spacecraft and the local space environment. These forces and torques can be used for a variety of novel touchless actuation concepts, such as towing space debris out of the geosynchronous orbit regime (GEO), and de-spinning uncontrolled spacecraft before servicing or docking. Modeling electrostatic forces and torques is vital to designing stable control laws to guarantee performance and avoid collision in a close formation flying context. Previous work for faster-than-realtime methods assumed the spacecraft was continuously conducting. In this paper, modeling electrostatic forces and torques on spacecrafts with electrically isolated dielectric regions are investigated by modifying the Multi-Sphere Method. This work is done using analytical as well as numerical methods.

INTRODUCTION

In the Geosynchronous Earth Orbit (GEO) regime, satellites charge to very high voltages on the order of ten kiloVolts.[1] This charging causes small forces and torques on the body due to interactions with earth's magnetic field, which changes the orbits of some uncontrolled lightweight debris objects through the Lorentz force.[2–5] If nearby spacecraft use active charging such as electron and ion guns, larger forces and torques are felt between the crafts. This enables novel Coulomb formation flying missions.[6–8] These forces can also be used for touchless re-orbiting of GEO debris to its graveyard orbit in a matter of months using the Electrostatic Tractor (ET).[9] If a spacecraft has a non-symmetric charge distribution, it also experiences torques which can be harnessed for touchless de-spin before servicing or grappling.[10–12]

There are many separate challenges to electrostatic actuation such as prescribing the appropriate electron and/or ion beam current and voltage, sensing the voltage, position, and attitude of a passive space object, and designing control laws that perform well for either tugging or de-spinning. In order to design and implement stable and performant control laws in any of the above mission scenarios, accurate and fast methods are needed to predict the force and torque on both spacecraft using only in-situ measurements such as the voltage of each craft, and their relative separation and attitude. This is important because under or over prediction can seriously harm performance, or lead to a collision.[13] This paper discusses how to predict electrostatic force and torque for a body that is composed of conductors and dielectrics.

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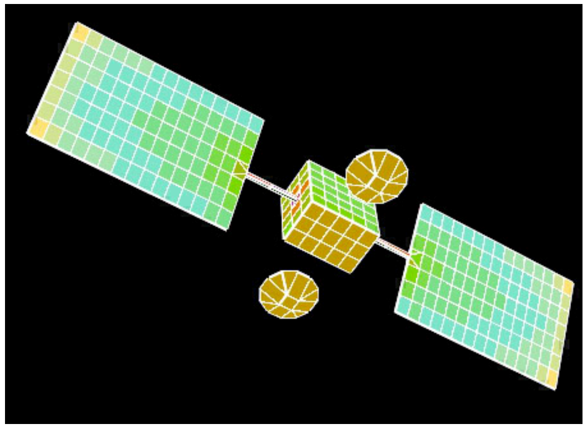


Figure 1. Surface voltages of a spacecraft with conducting and dielectric surfaces. Computation done in Nascap

There are many methods for force and torque prediction ranging from very accurate but much slower than realtime methods such as Finite Element Analysis (FEA) or faster and more scalable methods such as the Method of Moments (MoM)[14] or Boundary Element Method (BEM). Recent work explores using the Galerkin method to model forces between two dielectric spheres.[15] A relatively new scheme for electrostatic force and torque prediction is the Multi-Sphere Method (MSM).[16] MSM is very similar to the MoM in that is creates an elastance matrix which it inverts to find the charge distribution. The Coulomb force is applied from every discretized charge on one body to every discretized charge on the other body. MSM differs from MoM in that the elements of the elastance matrix are tuned to match force and torque created by a higher-fidelity method rather than from first principles. Because of this tuning, MSM can predict forces and torques with only a few percent error using only 3-4 spheres for a two craft system,[17,18] but requires a truth model to optimize from.

However, MSM has only been applied to continuously conducting spacecraft. Most spacecraft are built to be continuously conducting to avoid differential charging and arcing. However, some of the conducting covering may degrade with time and lose their conductivity. Two scenarios where this may occur is the coverglass coating on the solar panels and the Multi-Layer Insulation. Solar panels require a glass cover to protect from proton radiation, and there is usually a conductive clear coating over the glass, however, this coating may degrade or flake off in space and can leave sections of the non-conductive glass exposed. MLI also usually has a gold or aluminum coating, but this may flake off or otherwise degrade. Additionally, some spacecraft are not built fully conducting to begin with, and will have large dielectric portions. In the case of coverglass and MLI, there is a thin layer (10-100 μm) of dielectric sitting directly on top of a conductor connected to spacecraft ground. However, in an effort to save weight, some spacecraft have MLI wrapped around a skeleton frame. This complicates matters since part of the dielectric is directly connected to the conducting bus, while the rest is free floating. This case is not considered in this paper.

To estimate the amount of charge stored in a charged dielectric, approximate a complex spacecraft as a parallel plate capacitor. The first plate is the conducting spacecraft bus and the second plate is the surface of the dielectric, thus the separation is the thickness of the dielectric. The mutual capacitance is then $C_m = \frac{\epsilon_0 \epsilon_R A}{d} \approx 0.35$ μF/m^2 for a 1 mil thick sheet of MLI. Assume $\epsilon_R = 1$ for simple computation. The capacitor equation $Q = C_m \Delta V$ gives the difference in charge between the two plates. Assume the spacecraft bus is neutral so that half the charge is on each plate, making the surface charge density on the dielectric 0.17 μC/m^2 V. Thus, a 1 m^2 sheet of 1 mil thick dielectric charged to 200 V different than the spacecraft bus will hold approximately 35 μC of charge. There are many assumptions that go into this simple calculation – the self capacitance of the dielectric is ignored, and the conductor is assumed to be neutral. This calculation is meant to estimate the order of magnitude of the charge density, not to accurately predict it.

The amount of charge will increase linearly with the voltage difference, linearly with the area, and linearly with the relative permittivity. The stored charge increases inversely with the thickness of the dielectric, so doubling the thickness will halve the charge. From this, we conclude that surface charge densities on the order of 35 μC/m^2 are reasonable to assume during a harsh charging event. Keep in mind that an isolated fully conducting sphere of 1m radius will store only 1.1 μC when charged to 10 kV, much less than that stored on the dielectric.

The inclusion of dielectrics introduces two new questions: first, how do dielectrics charge? How much charge is there, and how is it distributed. The second question assumes knowledge of the first, and asks how to best model the inter-craft electrostatic forces quickly and accurately. This paper addresses the second question – Assuming knowledge of the voltage of the conducing spacecraft body and the surface charge density of all conductors, as well as the relative position and attitude, what are the force and torque on both bodies?

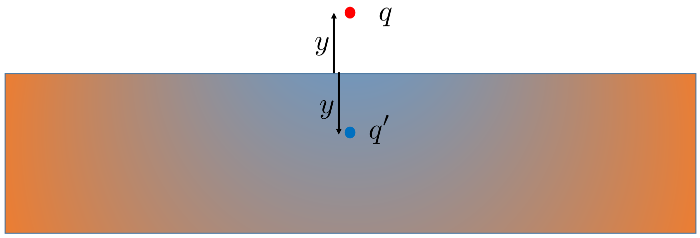
This is done by first solving a simpler problem using an analytical technique known as the method of images. Next, the Multi-Sphere Method (MSM) is introduced, and proposed modifications are made for including dielectrics. Lastly, three options for the inclusion of dielectrics for force and torque modeling with MSM are presented.

THE METHOD OF IMAGES

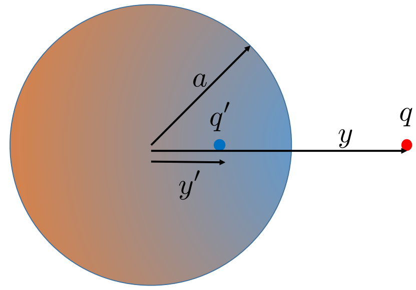
The method of images (MOI) is a trick for solving electrostatic boundary problems by placing point charges outside the boundary that artificially enforce the boundary conditions. It transforms electrostatic problems with boundary problems to problems without boundary conditions on a larger volume. Consider the case of a point charge q a distance y above an infinite grounded conducting plate as shown in Fig. 2(a). Charge of opposite polarity will be attracted to the point charge and will pool up below the point charge on the conductor. The system could be solved above the plane with a boundary condition of 0 V on the surface of the plane, or an imaginary image charge q' can be placed a distance y below the surface of the plane. If the image charge is opposite the original charge $q' = -q$, the surface of the plane will be at 0 V due to symmetry and the boundary condition does not have to be enforced. For all computations involving voltage, E field, or force, the imaginary image charge is just as good as the real surface charge distribution.

Now consider the system of a conducting sphere and point charge rather than an infinite plane, as shown in Fig. 2(b). For this system, the required image charge to keep the voltage of the surface at a specified voltage is less than the original charge ($q' = -\frac{a}{y}q$) and its position depends on the position of the original charge $y' = \frac{a^2}{y}$. As the point charge approaches the surface of the sphere, the image charge comes forward to meet it, and approaches the equal and opposite charge of the original. Good treatments of the MOI are found in.[19,20] To the author's knowledge, the MOI has only been applied to infinite planes, spheres, and point charges due to symmetry.

The MOI cannot solve the problem of a charged dielectric on the surface of a general shaped conductor, however, it can contribute some insights. As with the point charge near the surface of



(a) Infinate plane



(b) Sphere

Figure 2. Method of Images on two simple cases

a conducting sphere, an “image” charged dielectric can be thought of just on the inside of the conductor. For a small and thin dielectric on a large conducting body, the image charge will completely cancel out the original charge. For a larger, thicker dielectric, the image charge will be smaller, and they will be farther apart.

To put rough numbers to this, consider a conducting body of effective radius 1 meter and a dielectric of thickness 20μm that stores 35 muC of charge. The “image” dielectric will lie $R - \frac{R^2}{R+d} \approx 20\mu$m inside of the dielectric, and hold $-\frac{R}{R+d}q \approx -34.9\mu$C. The total charge contributed by the dielectric will be $q + q' \approx 0.7$ nC, a very small amount in comparison to either the charge stored on the dielectric or the conductor. For instance, 0.7 nC translates to $Q/C \approx 6.3$ V on the conductor. Even though the net charge is small, the separation may cause a significant force or torque in a non-flat electric field such as might be generated by a nearby charged spacecraft. Consider a $q_p = 1\mu$C point charge R =5 meters away, the total force caused by the dielectric and image charge will be

$$F = \frac{1}{4\pi\epsilon_0}\left(\frac{qq_p}{(R-d)^2} + \frac{q'q_p}{(R+d)^2}\right) \approx 31\ \mu\text{C} \tag{1}$$

Considering nominal forces $\sim$ 1 mC, this is a small but not insignificant change. As for torques, cross the force with a vector connecting the center of mass to the point about which the force is applied to get approximately T = 30 μCm, also a small but not insignificant fraction of the nominal torques. From this simple example, it seems that the inclusion of charged dielectrics should not change force and torque by a large amount since the image charge will almost cancel out the effects. In the case where the dielectric is far from the conductor, such as where MLI is wrapped directly around the bus, the effects will likely be more significant.

THE MULTI-SPHERE METHOD

Conducting MSM

The Multi-Sphere Method approximates a charged conductor as a collection of spheres as is shown in Fig. 3. All spheres are prescribed to be at the same voltage, and the elastance matrix is used to translate that voltage into charge. Once the charges are known, Coulomb’s law is used between every sphere in the tug and every sphere in the debris. This returns the electrostatic force and torque on both bodies.

The elastance matrix $[S]$ is formed from the size and positions of the spheres, which are tuned to

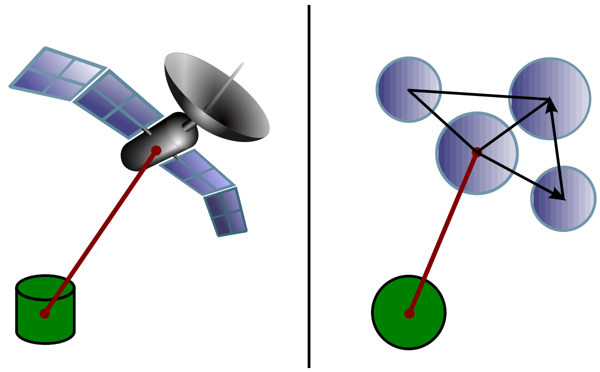


Figure 3. Approximation of a satellite as a collection of charged spheres.[16]

match force and torque. If the spheres are far from one another, the voltage of sphere i is given by:

$$V_i = \frac{1}{4\pi\epsilon_0}\frac{q_i}{R_i} + \sum_{j\neq i}^{N} \frac{1}{4\pi\epsilon_0}\frac{q_j}{\rho_{i,j}} \tag{2}$$

where q_j is the charge of the j^{th} sphere, $\rho_{i,j}$ is the distance between sphere i and sphere j, and R_i is the radius of the i^{th} sphere for a model with N spheres. If the voltages of each sphere are given by $\boldsymbol{V} = [V_1, V_2, ...V_m]^{\text{T}}$ and the charges are given by $\boldsymbol{q} = [q_1, q_2, ...q_m]^{\text{T}}$, the relationship between the two is

$$\boldsymbol{V} = [S]\,\boldsymbol{q} \tag{3}$$

To find the charges, the linear system is solved numerically. Equation (2) is used to write $[S]$ explicitly below in Eq. 4

$$[S] = \frac{1}{4\pi\epsilon_0}\begin{bmatrix} 1/R_1 & 1/\rho_{1,2} & \cdots & 1/\rho_{1,N} \\ 1/\rho_{2,1} & 1/R_2 & \cdots & 1/\rho_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\rho_{N,1} & 1/\rho_{N,2} & \cdots & 1/R_N \end{bmatrix} \tag{4}$$

Take note that this matrix is symmetric and strictly positive with the largest entries on the diagonal. Its inverse is the capacitance matrix $[C]$ and always has large positive entries on the diagonal and small negative entries everywhere else. If there are two conducting bodies, the matrix takes on a block form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} S_1 & S_M \\ S_M^T & S_2 \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tag{5}$$

Where V_1 and V_2 are vectors of the voltages at each sphere on bodies 1 and 2, Q_1 and Q_2 are the charges on those spheres. S_1 and S_2 are the elastance matrices as defined in Eq. (4) for body 1 and 2, and S_M is the mutual elastance matrix which is similar to the self elastance matrix, but has no $1/R$ terms since it is not on the diagonal, rather it only has the mutual terms linking the spheres from one body to the spheres on the other body. This linear matrix equation is numerically solved for the charge on each sphere.

Once the charges on each sphere are known, the forces and torques can be computed as shown in Eqs. (6) and (7). An origin O at the center of mass of the body is used for $\boldsymbol{r}_i$, where $\boldsymbol{r}_i$ is the vector that points from the origin to the i^{th} sphere. The force and torque calculated about this origin are then

$$\boldsymbol{F} = -k_c q_B \sum_{i=1}^{n} \frac{q_i}{{r_{i,b}}^3} \boldsymbol{r}_{i,b} \tag{6}$$

$$\boldsymbol{L}_O = -k_c q_B \sum_{i=1}^{n} \frac{q_i}{{r_{i,b}}^3} \boldsymbol{r}_i \times \boldsymbol{r}_{i,b} \tag{7}$$

In other methods such as MoM or the BEM, the self and mutual elastances would be derived from first principles. In this method they are tuned via adjusting the radius R_i and position $\boldsymbol{r}_i$ of the spheres. This is typically done using a numerical optimizer which is allowed to vary the size and positions of the spheres with optional constrains based on symmetry to minimize a cost function built from the differences between the force predicted by the MSM model and a truth model at a set of sample points.

Proposed Modifications for Dielectric MSM

In a conductor, charge will move so that the electric potential energy is at a minimum. This also means that every location on a conductor has the same voltage, even if more or less charge is required to hold that voltage due to mutual interactions. Because the natural charging time for a fully conducting spacecraft is so small, the environmental currents are easily able to provide the extra charge. The natural charging time of a conducting spacecraft is proportional to the self capacitance, here typically $\sim$100 pF.

However, many of the dielectrics used on satellites are very thin, which creates much larger *mutual* capacitances values near $\sim$ 100 nF. This creates a longer natural charging time, which do not allow charge to move on and off fast enough to hold the same voltage. Thus, while conductors can be thought of as lumped *voltages*, dielectrics can be thought of as lumped *charges*. This paradigm, treating the conductors and dielectrics separately, is used to modify MSM to include dielectrics.

The fundamental concept of MSM, that the voltage of every model sphere is a function of its own charge as well as all neighboring spheres is preserved. What changes is that rather than assuming knowledge of all voltages and solving for all charges, now the voltages of all *conducting* spheres and the charge of all *dielectric* spheres is assumed to solve for the charges of all conducting spheres. The charges on the conducting spheres are appended to the apriori known dielectric charges, and the coulomb force and torque can be computed.

To come up with an augmented equation, consider an isolated spacecraft with N conducting spheres and M dielectric point charges.

$$\begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} S_{1,1} & \dots & S_{1,N} & S_{1,N+1} & \dots & S_{1,N+M} \\ \vdots & & & & & \vdots \\ S_{N,1} & \dots & S_{N,N} & S_{N,N+1} & \dots & S_{N,N+M} \end{bmatrix} \begin{bmatrix} Q_1 \\ \vdots \\ Q_N \\ Q_{N+1} \\ \vdots \\ Q_{N+M} \end{bmatrix} \tag{8}$$

$$= [S_C]\boldsymbol{Q}_C + [S_D]\boldsymbol{Q}_D \tag{9}$$

Where the C or D subscript indicates conductor or dielectric, respectively. To solve for the unknown $\boldsymbol{Q}_C$, rearrange the equation to yield:

$$[S_C]\boldsymbol{Q}_C = \boldsymbol{V} - [S_D]\boldsymbol{Q}_D \tag{10}$$

which can be solved numerically. Keep in mind that this method assumes knowledge of the voltage of the spacecraft bus as well as the surface charge density or total charge on the dielectrics. For a two craft system, both the $[S_C]$ and $[S_D]$ matrices divide into blocks just as in Eq. (5).

The engineer then tunes the size and location of the spheres, which affect the $[S_C]$ and $[S_D]$ matrices. This changes the charges, which change the force and torque. The parameters are tuned so as to produce the same force and torque as a high fidelity truth model.

Dielectric MSM test case

Surface MSM (SMSM) is a variant of MSM that uses many equal radius spheres placed equidistantly on the surface of the conductor.[21] Once the positions of all spheres are found, their radius is varied so that the self capacitance of the SMSM model matches that of the physical object, which can be calculated analytically or using a high fidelity FEA program. For dielectrics, the formulation introduced above is used and a sheet of points rather than spheres is placed very close to the surface of the conductor. The spacing between neighboring conducting spheres is used as the separation between conductor surface and dielectric points, but this value could be optimized at a later time.

Consider the special case of a conducting cylinder along the y axis with a diameter of 1 meter and length of 3 meters. This prototype conducting shape has been analyzed in many contexts,[16–18,21,22] however a thin dielectric is now added to the positive y end. A SMSM model with 1041 nodes (spheres and point charges) is shown in Fig. 4. The conducting body is charged to 30 kV, and the dielectric is given a charge density of 35 μC/m^2 which translates to 27 μC of total charge. For comparison, the conducting cylinder when charged to 30 kV without the dielectric holds just over 3 μC.

From this model, moments of the charge distribution can be calculated. It has been found that the first two measures, the total charge Q and the electric dipole $\boldsymbol{q}$ are excellent proxies for the force and torque on a body.[23] The total charge is simple to understand, and the electric dipole is the first moment of the charge distribution. Both are defined for the continuum and MSM case for a MSM model with N conducting spheres and M point charges in Eq. (11):

$$Q = \int_B \mathrm{d}q = \sum_{i=1}^{N} q_{c_i} + \sum_{i=1}^{M} q_{d_i} \qquad \boldsymbol{q} = \int_B \boldsymbol{r}\mathrm{d}q = \sum_{i=1}^{N} \boldsymbol{r}_i q_{c_i} + \sum_{i=1}^{M} \boldsymbol{r}_i q_{d_i} \tag{11}$$

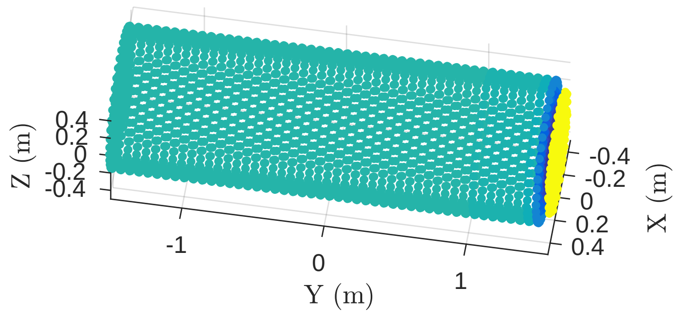


Figure 4. SMSM model of cylinder with charged dielectric end cap. Color indicates charge

As with any discretized numerical model, the outputs will only be valid for a high enough resolution. To determine that resolution, the number of nodes is varied from 117 to 2544 in 39 steps and the total charge and electric dipole are calculated at each step. The only changing input is the resolution and the goal is to find the resolution at which the solution is a straight line and has no more dependance on the resolution. This is done in Fig. 5

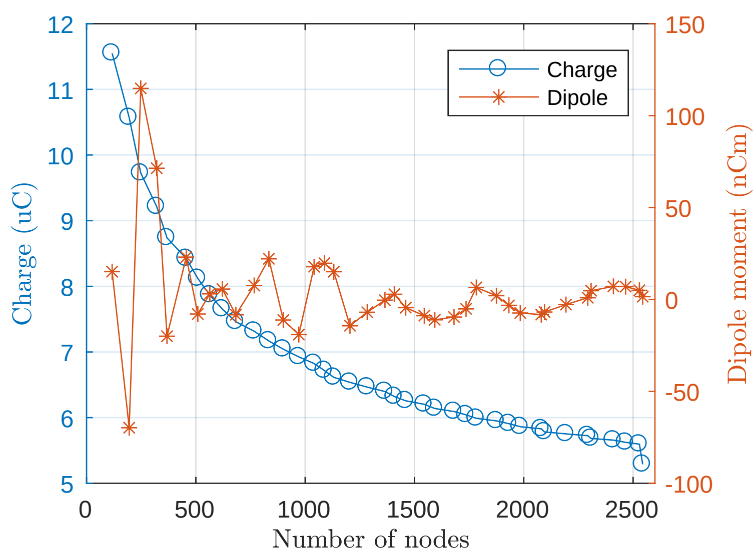


Figure 5. Sensitivity of outputs to number of nodes

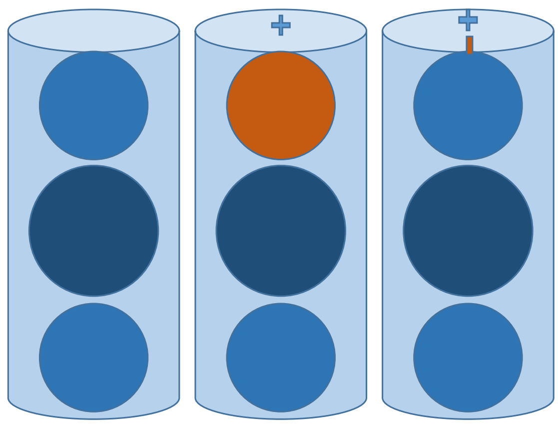
The total charge is plotted with circles on the left axis, and the electric dipole is plotted with asterisks on the right. The total charge is a smooth curve with a decreasing slope. It appears that while the total charge still has some dependency on resolution at this scale, it will eventually become flat if enough spheres are used. At the relatively coarse initial point of 117 nodes, the total charge is near 12 uC, and at the relatively fine resolution of 2544 nodes it is near 5 uC. The MOI would predict that the net charge contributed by the dielectric is nearly zero, which would say that the charge should plateau at around 3.1 μC. The electric dipole, in contrast, takes a lot longer to plateau, and is not monotonic.

Increasing the resolution beyond this point is very computationally demanding, since the brute force method to invert a matrix of size $N \times N$ increases with N^3. To solve the system with only 117 nodes takes 0.6 seconds, while with 2544 nodes it takes more than 3 minutes. This extra time makes it infeasible to use this in dynamical control laws as it is much slower than realtime.

This analysis confirms some of the intuition gained from the MOI analysis. Charge of the opposite polarity does indeed pool up beneath a charged dielectric, and cancels out a lot of the charge from the dielectric. SMSM and MOI do disagree on just how much of that charge is canceled out, although they might agree better if more spheres are used.

FORCE AND TORQUE MODELING

In this section, three possible approaches are presented for modeling force and torque on a conductor with a charged dielectric. The three approaches are shown pictorially in Fig. 6



(a) Ignore (b) Point Charge (c) Charge Pair

Figure 6. Three proposed methods for including dielectrics in MSM

Modeling of Conductor

The first and easiest method, shown in Fig. 6(a) is to completely ignore the dielectric. For a small dielectric in direct contact with the spacecraft bus, the image charge may cancel out enough of the charge that the force and torque changes are negligible. This method will be most applicable for thin dielectrics sitting directly on the conducting spacecraft surface.

Isolated Charge

The second method, shown in Fig. 6(b), would be to add one or more point charges where the dielectric physically is. Since the dielectric's position and charge are assumed known, the point charge could be placed without any optimization. The conducting portion of the MSM model could still be optimized though. If more accuracy is needed, the point charge could be allowed to move slightly, and change its charge. This approach would make the nearest sphere in the conducting MSM model take on a charge roughly equal to the image charge.

Charge Pairs

The third option, shown in Fig. 6(c), would be to add in a pair of point charges for each dielectric present in the system. Since MOI indicates that a static charge exists beneath the surface of the conductor, why not put it there to begin with? As with the second option, the position and magnitude of both point charges could be held fixed at values predicted by MOI, or optimized over. Using pairs of charge that mostly cancel each other out would allow pre-exiting methods for placing MSM spheres inside a conductor to still be used with minimal changes.

CONCLUSION

This paper is a first step towards modeling the complex problem of electrostatic actuation on conductors and dielectrics. This is done first by estimating the amount of charge stored on dielectrics due to likely voltage differences. Next the Method of Images is used to gain intuition into the problem, and it is found that it is likely that for the case of small, thin dielectric the image charge may completely cancel out the effects of the dielectric. Next MSM is introduced, and a SMSM model of a test case is presented. This test case shows a large pooling up of charge underneath the dielectric that cancels out most of the dielectric's charge. However, SMSM predicts that the dielectric has more of an impact than MOI does, but this may be due to grid size. Lastly, three new methods for modifying MSM to account for dielectrics are presented: ignoring the dielectric, modeling the dielectric as a point charge, or modeling the dielectric as a pair of point charges. Future work will investigate which of these options predicts the force and torque with the best accuracy using a small number of spheres

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