Spacecraft Formations Dynamics Using Variational Equations of First-Order Relative Motion Invariants

Trevor Bennett
Graduate Research Assistant

Hanspeter Schaub
Alfred T. and Betty E. Look Professor of Engineering

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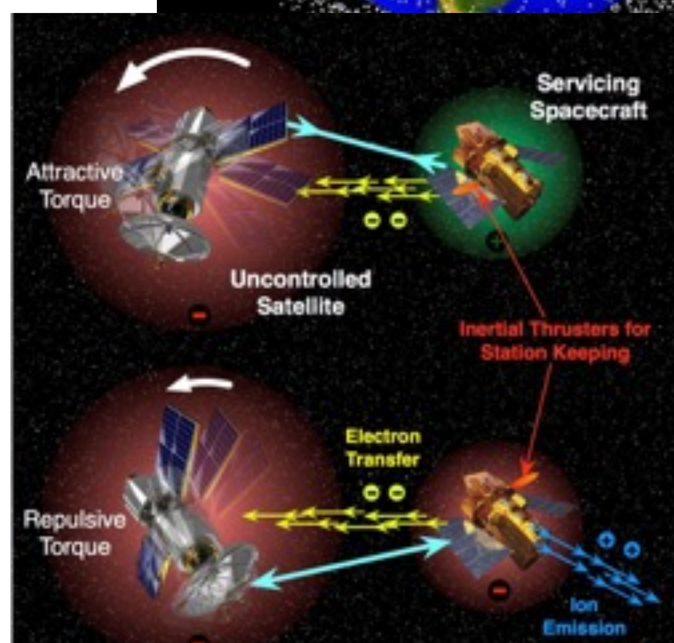
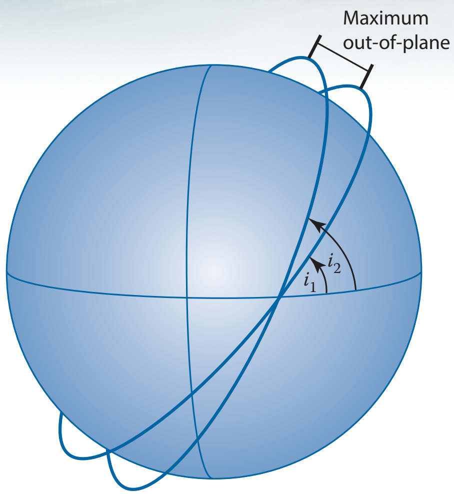
Motivation

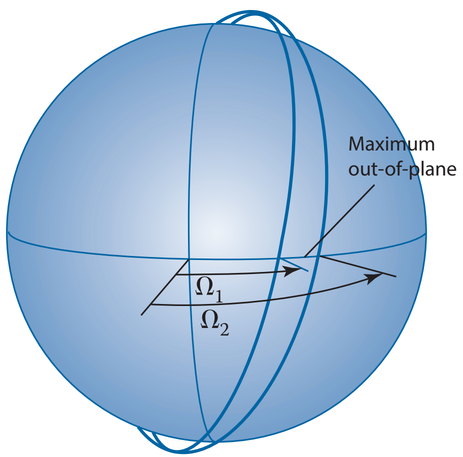
Objective:
Utilize simple **geometrically insightful** parameters for relative orbit reconfiguration.



Orbit Element Difference Kinematics

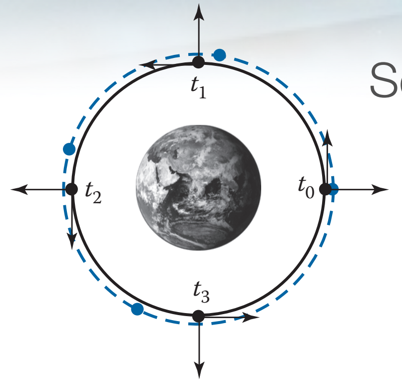


Inclination

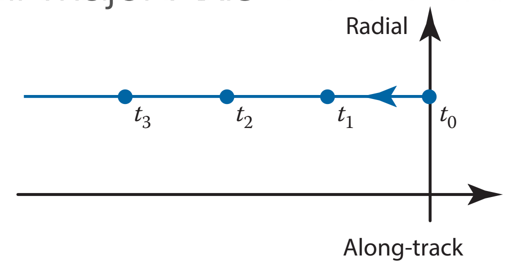


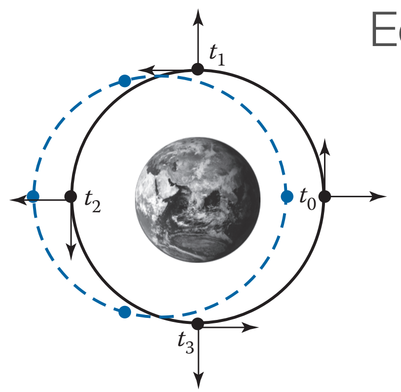
Ascending Node

Schaub, H., Vadali, S. R., and Alfriend, K. T., “Spacecraft Formation Flying Control Using Mean Orbit Elements,” *Journal of the Astronautical Sciences*, Vol. 48, No. 1, 2000, pp. 69–87.

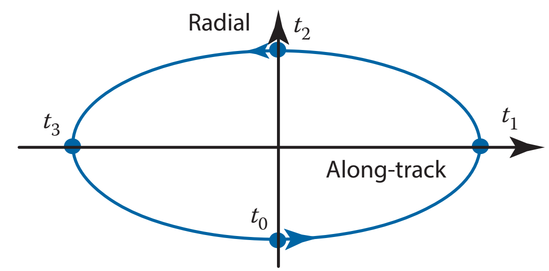


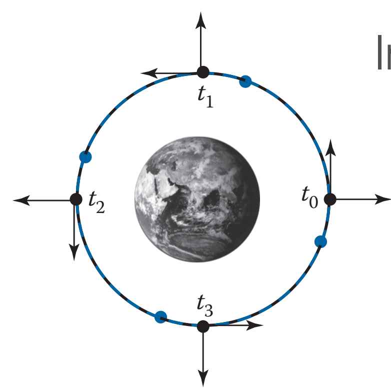
Semi-Major Axis



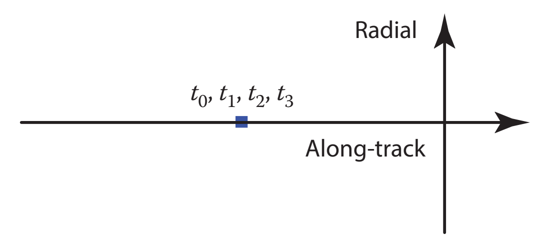


Eccentricity

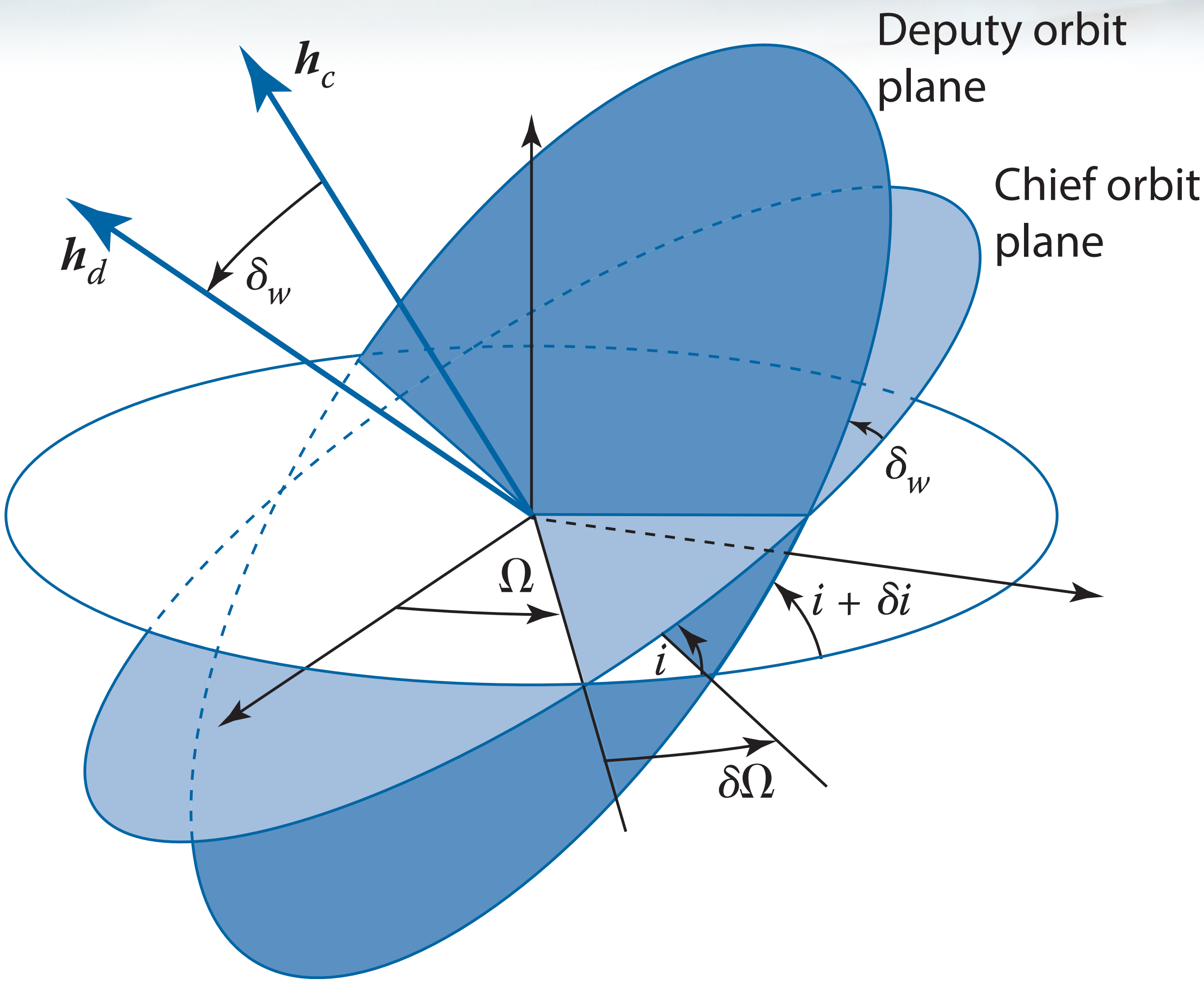




Initial Anomaly Angle



Eccentricity/Inclination Vector Difference Kinematics



$$\vec{e} = \begin{pmatrix} e_X \\ e_Y \end{pmatrix} = e \cdot \begin{pmatrix} \cos\omega \\ \sin\omega \end{pmatrix}$$

$$\Delta\vec{e} = \vec{e}_2 - \vec{e}_1 = \delta e \cdot \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

$$\Delta\vec{i} \approx \begin{pmatrix} \Delta i \\ \sin i \Delta\Omega \end{pmatrix}$$

S. D. Amico, J. S. Ardaens, and R. Larsson, “In-flight demonstration of formation control based on relative orbit elements,” *4th International Conference on Spacecraft Formation Flying Missions and Technologies*, August 18-20 2011.

Montebruck, O., Kirschner, M., and D’Amico, S., “*E/I*-Vector separation for grace proximity operations,” DLR/GSOC TN 04-08, 2004.

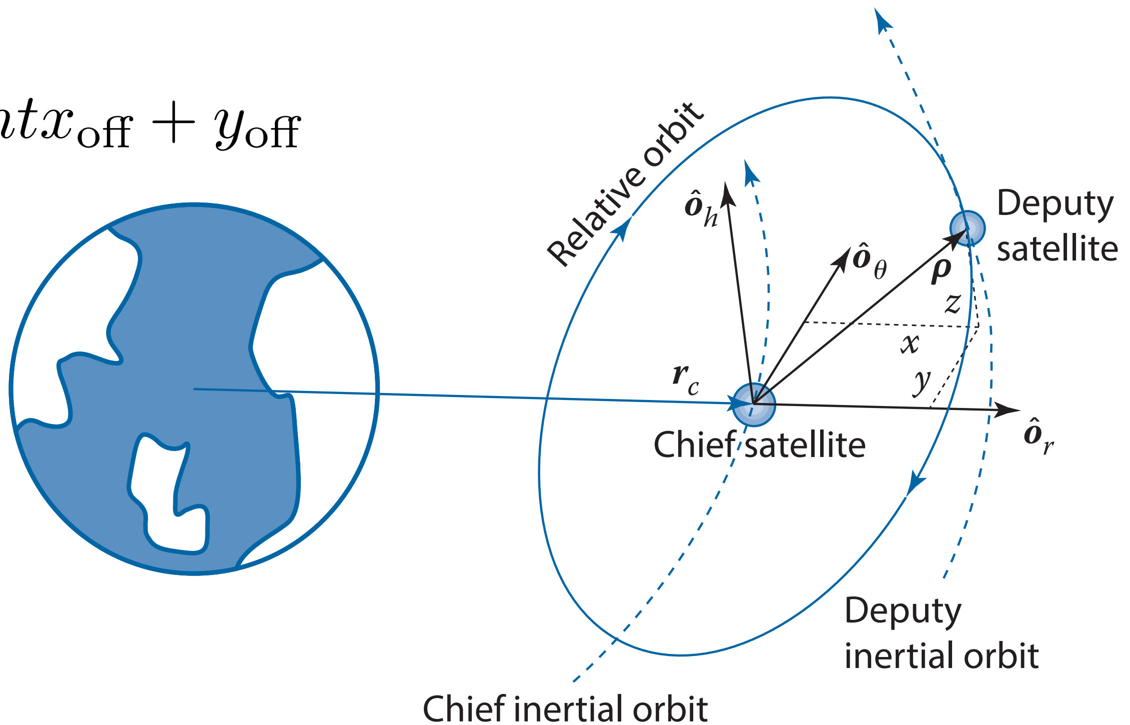
CWH Relative Motion Solution

Schaub, H. and Junkins, J. L., *Analytical Mechanics of Space Systems*, AIAA Education Series, Reston, VA, 2003.

$$x(t) = A_0 \cos(nt + \alpha) + x_{\text{off}}$$

$$y(t) = -2A_0 \sin(nt + \alpha) - \frac{3}{2} n t x_{\text{off}} + y_{\text{off}}$$

$$z(t) = B_0 \cos(nt + \beta)$$



Lovell, T. A. and Tragesser, S. G., "Guidance for Relative Motion of Low Earth Orbit Spacecraft Based on Relative Orbit Elements," *AIAA/AAS Astrodynamics Specialist Conference*, Providence, RI, Aug. 16–19 2004, Paper No. AIAA 2004-4988.

Lovell, T. A. and Spencer, D. A., "Relative Orbital Elements Formulation Based upon the Clohessy-Wiltshire Equations," Journal of Astronautical Sciences, 2015, pre-release available online, doi:10.1007/s40295-014-0029-6.

Gauss' Variational Equations

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2}{h}\left(e \sin f\, u_r + \frac{p}{r} u_\theta\right)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1}{h}(p \sin f\, u_r + ((p+r)\cos f + re)u_\theta)$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r\cos\theta}{h} u_h$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{r\sin\theta}{h\sin i} u_h$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{he}[-p\cos f\, u_r + (p+r)\sin f\, u_\theta] - \frac{r\sin\theta\cos i}{h\sin i} u_h$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{\eta}{he}[(p\cos f - 2re)u_r - (p+r)\sin f\, u_\theta]$$

LROE Variation Equations

$$x(t) = A_0 \cos(nt + \alpha) + x_{\text{off}}$$
$$y(t) = -2A_0 \sin(nt + \alpha) - 1.5nt x_{\text{off}} + y_{\text{off}}$$
$$z(t) = B_0 \cos(nt + \beta)$$

$$\mathbf{e} = \begin{bmatrix} A_0 \\ \alpha \\ B_0 \\ \beta \\ x_{\text{off}} \\ y_{\text{off}} \end{bmatrix}$$

LROE Invariant set

Singular ROE set where
α is ambiguous if $A_0 = 0$, or
β is ambiguous if $B_0 = 0$

Non-Singular LROE Set

$$A_1 = A_0 \cos(\alpha) \qquad A_2 = A_0 \sin(\alpha) \qquad B_1 = B_0 \cos(\alpha) \qquad B_2 = B_0 \sin(\alpha)$$

$$x(t) = A_1 \cos(nt) - A_2 \sin(nt) + x_{\text{off}}$$
$$y(t) = -2A_1 \sin(nt) - 2A_2 \cos(nt) - \frac{3}{2} nt x_{\text{off}} + y_{\text{off}}$$
$$z(t) = B_1 \cos(nt) - B_2 \sin(nt)$$

LROE Variation Equations

$$\boldsymbol{X} = (A_1, A_2, B_1, B_2, x_{\text{off}}, y_{\text{off}}) \qquad [L] = \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{e}}^T [J] \frac{\partial \boldsymbol{s}}{\partial \boldsymbol{e}} \qquad \dot{\boldsymbol{e}} = [L]^{-1} \left[\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{e}}\right]^T \boldsymbol{a}_d$$

$$\boxed{\dot{\boldsymbol{X}} = \frac{1}{n} \underbrace{\begin{bmatrix} -\sin(nt) & -2\cos(nt) & 0 \\ -\cos(nt) & 2\sin(nt) & 0 \\ 0 & 0 & -\sin(nt) \\ 0 & 0 & -\cos(nt) \\ 0 & 2 & 0 \\ -2 & 3nt & 0 \end{bmatrix}}_{[B(\boldsymbol{X},t)]} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}}$$

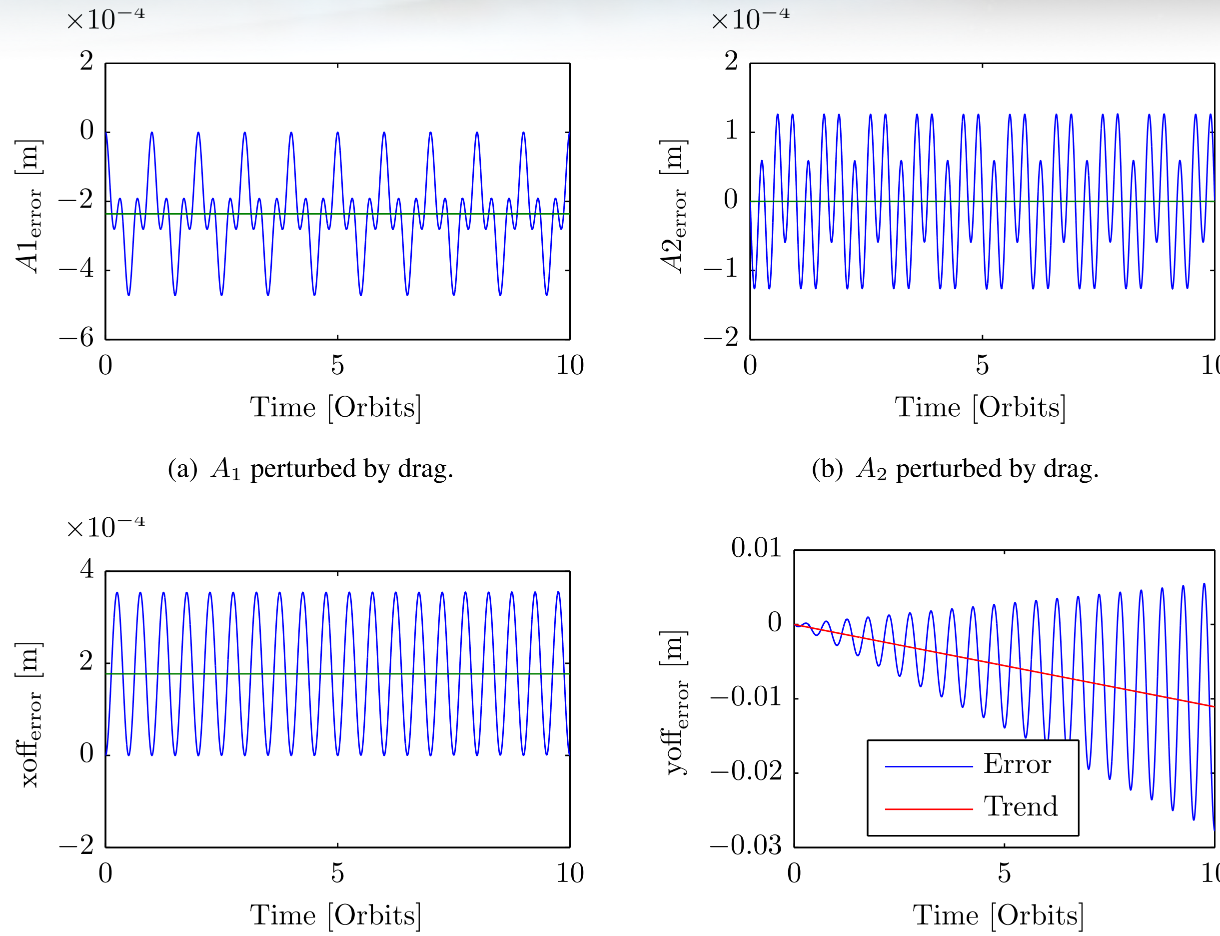
Bennett, T. and Schaub, H., "Continuous-Time Modeling and Control Using Linearized Relative Orbit Elements," *AAS/AIAA Astrodynamics Specialist Conference*, Vail, Colorado, Aug. 9–13 2015, Paper AAS 15–773.

Atmospheric Drag Illustration

$$\ddot{\boldsymbol{r}} = -\frac{1}{2} C_D \frac{A}{m} \rho_A \|\boldsymbol{V}_A\| \boldsymbol{V}_A$$

$$\boldsymbol{e}_r = \begin{bmatrix} A_{1,r} \\ A_{2,r} \\ B_{1,r} \\ B_{2,r} \\ x_{\text{off},r} \\ y_{\text{off},r} \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [\text{m}]$$

in-plane 2:1 ellipse



(a) A_1 perturbed by drag.

(b) A_2 perturbed by drag.

(c) x_{off} perturbed by drag.

(d) y_{off} perturbed by drag.

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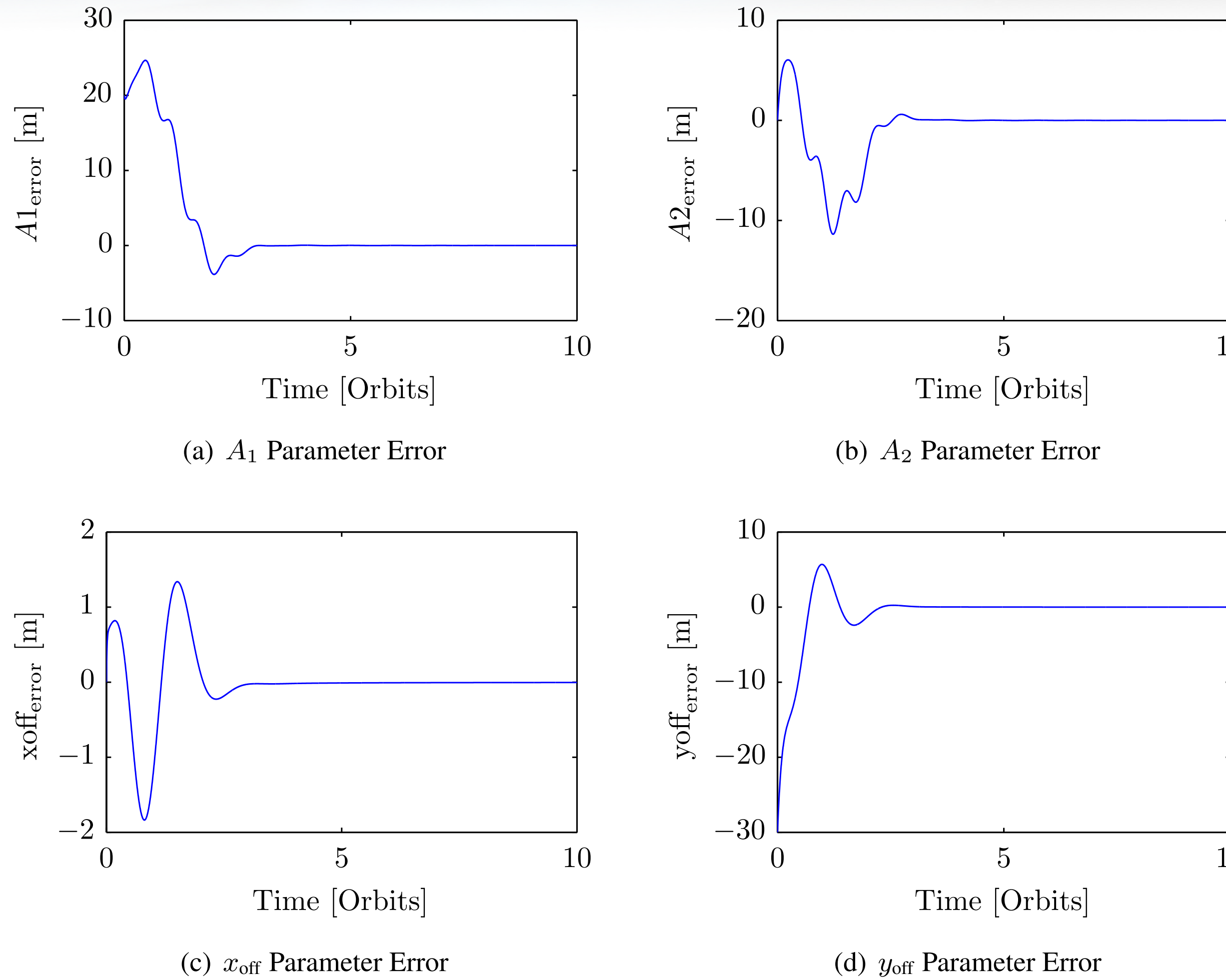
LROE Feedback Example
Ellipse to Lead-Follower

$$\boldsymbol{u} = -([B]^T[B])^{-1}[B]^T[K]\Delta\boldsymbol{e}$$

$$\boldsymbol{e}_0 = \begin{bmatrix} A_{1,0} \\ A_{2,0} \\ B_{1,0} \\ B_{2,0} \\ x_{\text{off},0} \\ y_{\text{off},0} \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [\text{m}]$$

$$\boldsymbol{e}_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 30 \end{bmatrix} \quad [\text{m}]$$



(a) A_1 Parameter Error
(b) A_2 Parameter Error
(c) x_{off} Parameter Error
(d) y_{off} Parameter Error

Bennett, T. and Schaub, H., "Continuous-Time Modeling and Control Using Linearized Relative Orbit Elements," *AAS/AIAA Astrodynamics Specialist Conference*, Vail, Colorado, Aug. 9–13 2015, Paper AAS 15–773.

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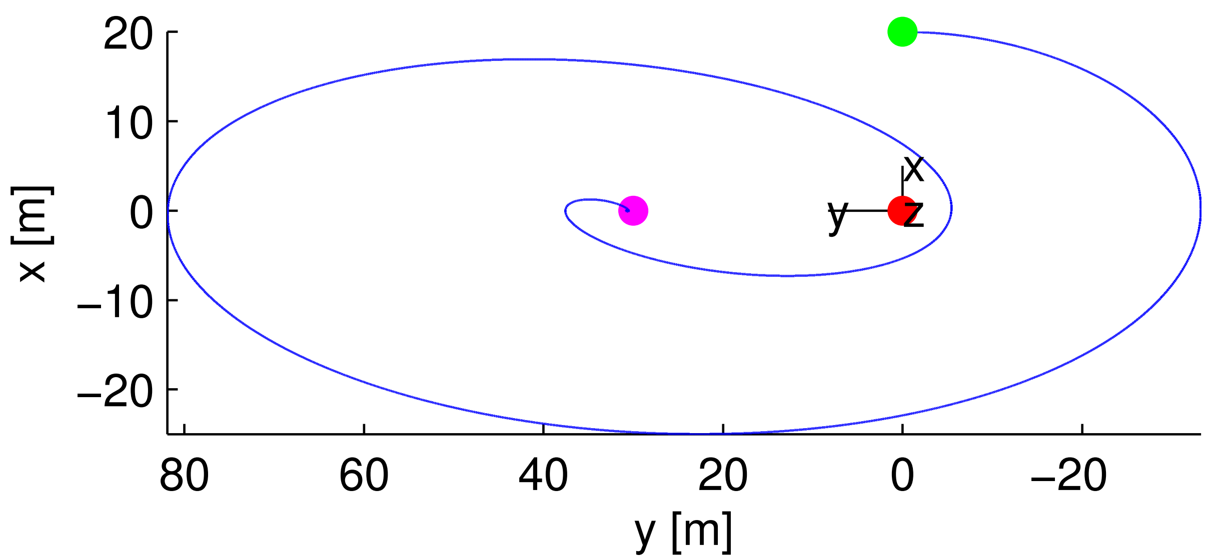

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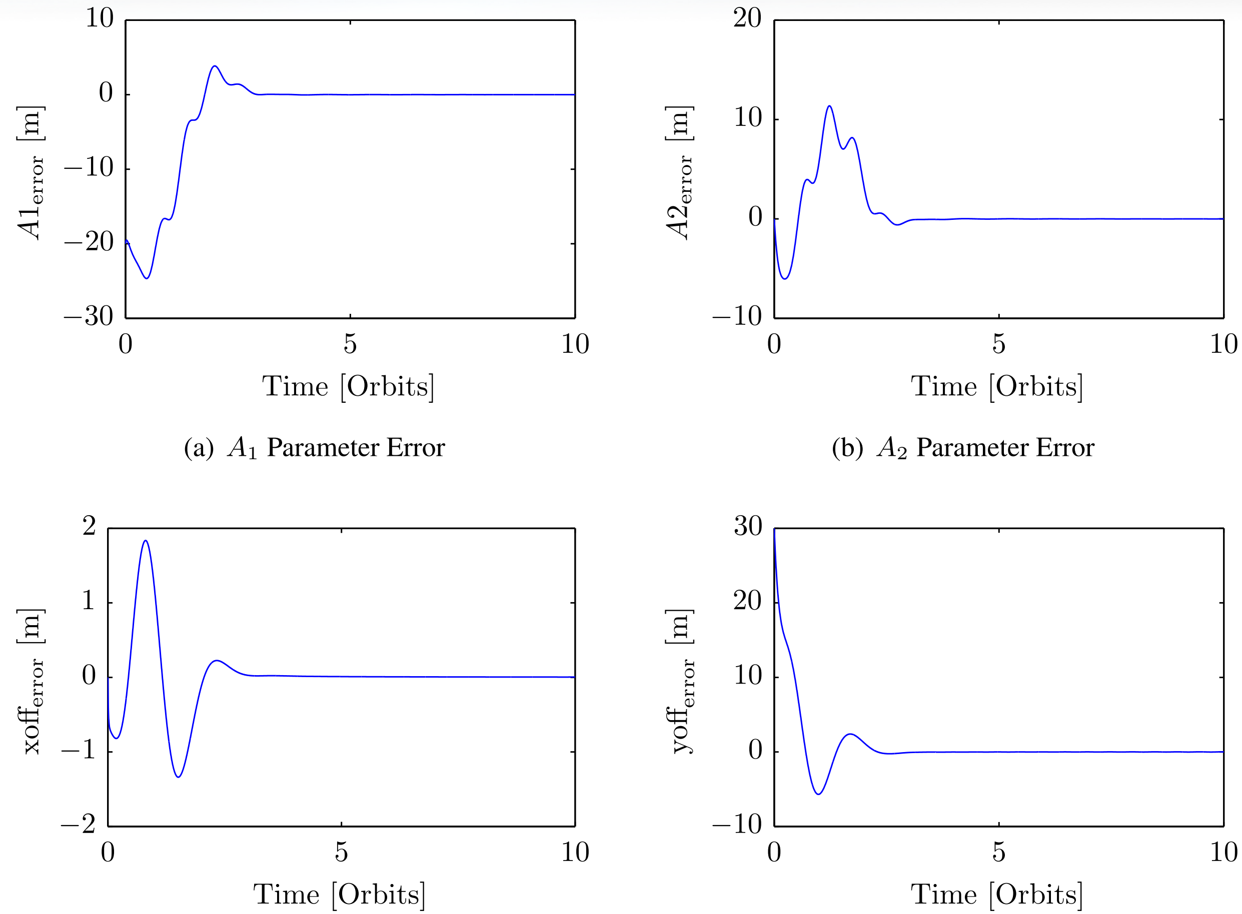

(a) A_1 Parameter Error

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(c) x_{off} Parameter Error

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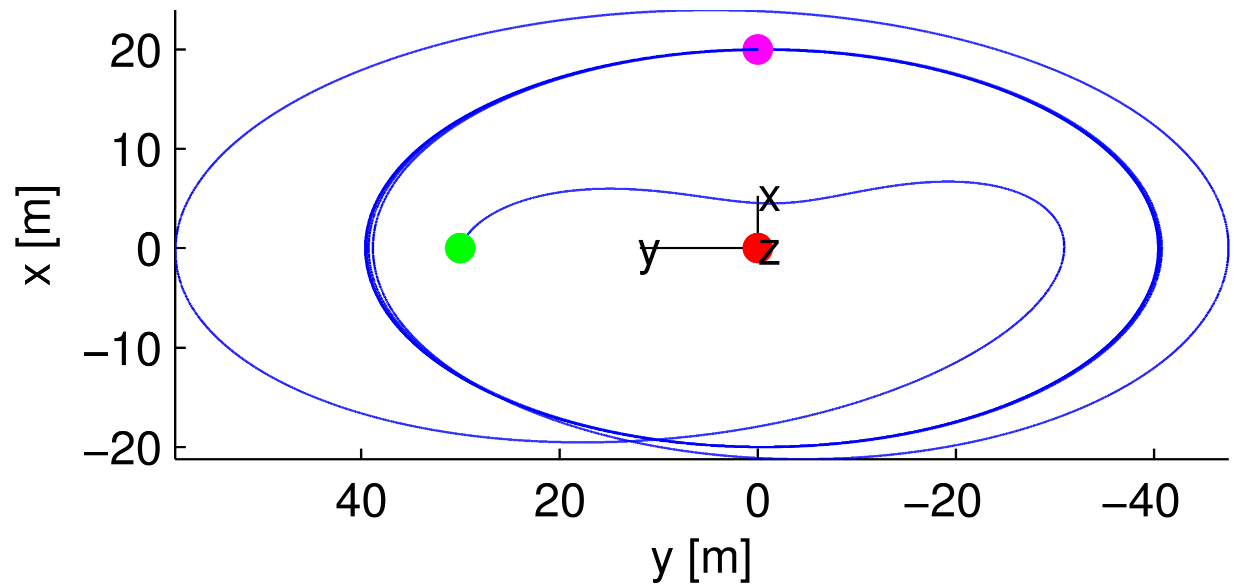

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Bearings-Only LROE Estimation

$$\boldsymbol{X} = (A_1, A_2, B_1, B_2, x_{\text{off}}, y_{\text{off}})$$

Not fully observable

$$\hat{\boldsymbol{X}} = \frac{1}{A_1}\begin{bmatrix} A_2 \\ B_1 \\ B_2 \\ x_{\text{off}} \\ y_{\text{off}} \end{bmatrix} = \begin{bmatrix} \hat{A}_2 \\ \hat{B}_1 \\ \hat{B}_2 \\ \hat{x}_{\text{off}} \\ \hat{y}_{\text{off}} \end{bmatrix}$$

Reduced non-dimensional LROE set

$$\hat{x}(t) = \cos(nt) - \hat{A}_2 \sin(nt) + \hat{x}_{\text{off}}$$

$$\hat{y}(t) = -2\sin(nt) - 2\hat{A}_2\cos(nt) - \frac{3}{2}nt\hat{x}_{\text{off}} + \hat{y}_{\text{off}}$$

$$\hat{z}(t) = \hat{B}_1\cos(nt) - \hat{B}_2\sin(nt)$$

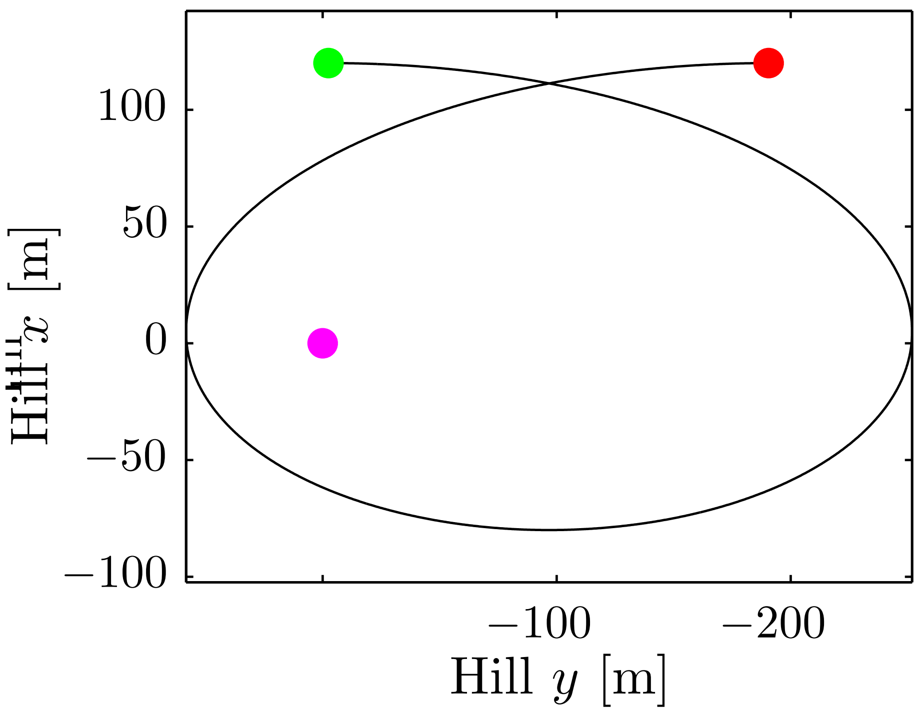
Non-dimensional CW solution

Bennett, T. and Schaub, H., "Space-to-Space Based Relative Motion Estimation Using Direct Relative Orbit Parameters," *Advanced Maui Optical and Space Surveillance (AMOS) Technologies Conference*, Wailea, Maui, Hawaii, Sept. 15–18 2015.

Angles-Only RLOE Example

$$\boldsymbol{X}^{\text{true}} = \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \\ x_{\text{off}} \\ y_{\text{off}} \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 200 \\ 0 \\ 20 \\ -2.5 \end{bmatrix} \text{ [m]}$$

$$\Delta\boldsymbol{X} = \begin{bmatrix} 10 \\ -2 \\ -7 \\ 2 \\ 5 \\ -5 \end{bmatrix} \text{ [m]}$$



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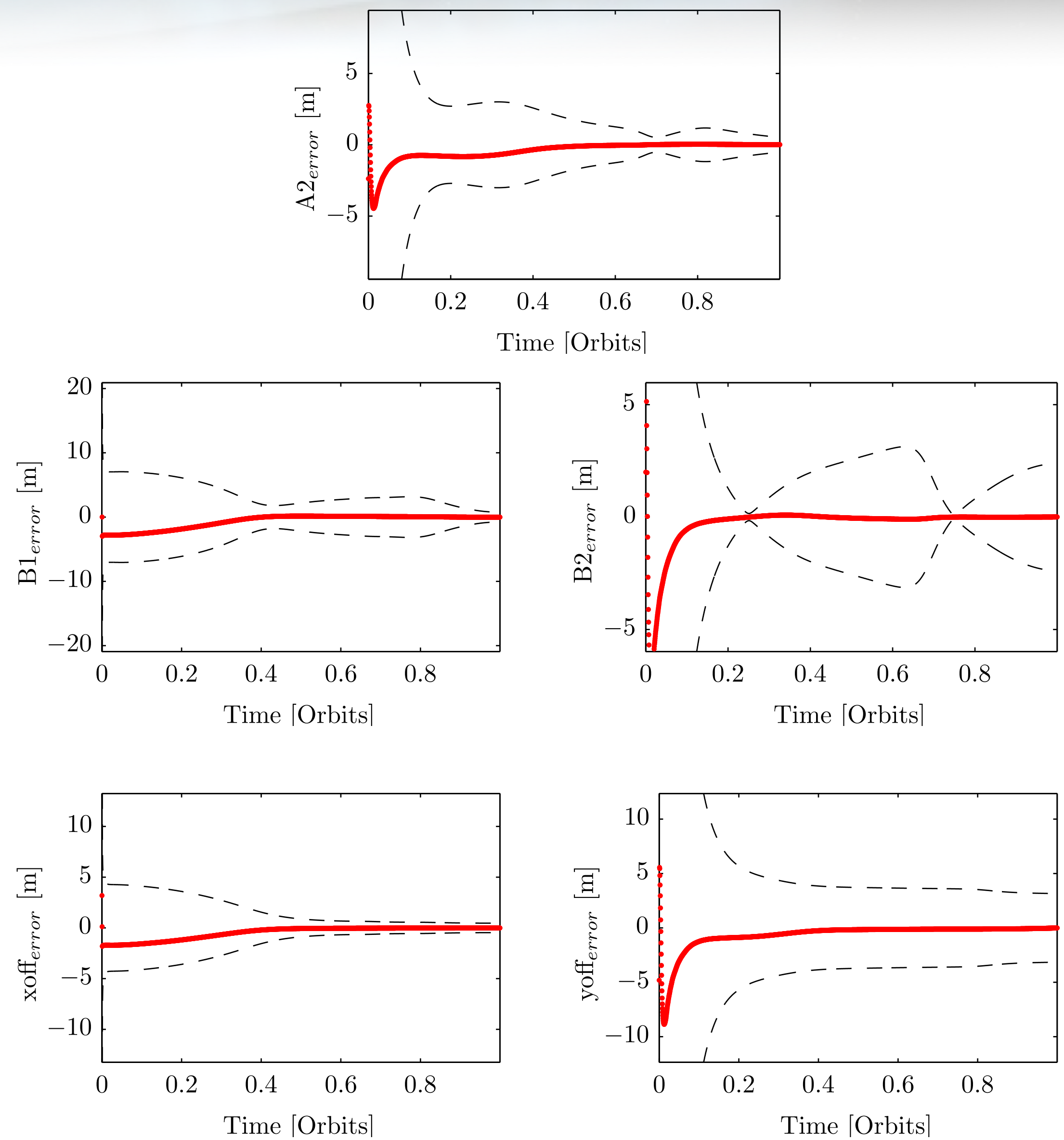
$$\Delta \boldsymbol{X} = \begin{bmatrix} 10 \\ \text{-}2 \\ \text{-}7 \\ 2 \\ 5 \\ \text{-}5 \end{bmatrix} \quad [\text{m}]$$



Conclusions

- LROE's form a geometrically insightful relative orbit descriptions
- Can simplify the relative orbit control formulation is particular formation characteristics are controlled
- Has shown promise in relative orbit estimation as well.
- Future work:
  - Apply perturbation forces directly to LROE formation to quantify accuracy of formation shape perturbation predictions
  - Expand LROE formulation from rectilinear to curvilinear coordinates
  - Investigate impulsive LROE control formulations based on the LROE variational equations

Questions?